

Proportion

Things to remember:

- Start by checking the question for squares, cubes and roots;
- "x is directly proportional to y" looks like $x \propto y$ or $x = ky$
- "x is inversely proportional to y" looks like $x \propto \frac{1}{y}$ or $x = \frac{k}{y}$

Questions:

1. The shutter speed, S , of a camera varies inversely as the square of the aperture setting, f .
When $f = 8$, $S = 125$

(a) Find a formula for S in terms of f .

$$S = \frac{k}{f^2}$$

$$125 = \frac{k}{64} \Rightarrow k = 125 \times 64 = 768$$

$$S = \frac{768}{f^2}$$

(3)

(b) Hence, or otherwise, calculate the value of S when $f = 4$

$$f = 4 \quad S = \frac{768}{16} = 48$$

$$S = 48$$

(1)

(Total 4 marks)

2. d is directly proportional to the square of t .
 $d = 80$ when $t = 4$

(a) Express d in terms of t .

$$d = kt^2$$

$$80 = k \times 16$$

$$5 = k$$

$$d = 5t^2$$

(3)

(b) Work out the value of d when $t = 7$

$$d = 5 \times 7^2$$

$$d = 245$$

(1)

(c) Work out the positive value of t when $d = 45$

$$45 = 5t^2$$

$$9 = t^2$$

$$3 = t$$

$$t = 3$$

(2)

(Total 6 marks)

3. The distance, D , travelled by a particle is directly proportional to the square of the time, t , taken. When $t = 40$, $D = 30$
- (a) Find a formula for D in terms of t .

$$D = Kt^2$$

$$30 = K \times 40^2$$

$$\frac{30}{1600} = K$$

$$D = \frac{3t^2}{160}$$

$$D = \dots\dots\dots$$

(3)

- (b) Calculate the value of D when $t = 64$

$$t = 64 \quad D = \frac{3 \times 64^2}{160}$$

$$76.8$$

$$\dots\dots\dots$$

(1)

- (c) Calculate the value of t when $D = 12$
Give your answer correct to 3 significant figures.

$$12 = \frac{3t^2}{160}$$

$$1920 = 3t^2$$

$$t^2 = 640$$

$$25.3$$

$$\dots\dots\dots$$

(2)

(Total 6 marks)

4. The time, T seconds, it takes a water heater to boil some water is directly proportional to the mass of water, m kg, in the water heater. When $m = 250$, $T = 600$

- (a) Find T when $m = 400$

$$T = km$$

$$600 = 250k$$

$$\frac{600}{250} = k$$

$$k = \frac{12}{5} = 2.4$$

$$T = \frac{12m}{5} \text{ or } 2.4m$$

$$m = 400$$

$$T = \dots\dots\dots 960$$

(3)

The time, T seconds, it takes a water heater to boil a constant mass of water is inversely proportional to the power, P watts, of the water heater.

When $P = 1400$, $T = 360$

- (b) Find the value of T when $P = 900$

$$T = \frac{k}{P}$$

$$360 = \frac{k}{1400}$$

$$\frac{504000}{1400} = k$$

$$P = 900$$

$$T = \dots\dots\dots 560$$

(3)

(Total 6 marks)

5. A ball falls vertically after being dropped.
 The ball falls a distance d metres in a time of t seconds.
 d is directly proportional to the square of t .
 The ball falls 20 metres in a time of 2 seconds.
 (a) Find a formula for d in terms of t .

$$d = kt^2$$

$$20 = k \times 4$$

$$5 = k$$

$$d = 5t^2 \dots\dots\dots$$

(3)

- (b) Calculate the distance the ball falls in 3 seconds.

$$5 \times 9$$

$$45 \dots\dots\dots \text{m}$$

(1)

- (c) Calculate the time the ball takes to fall 605 m.

$$605 = 5t^2$$

$$\div 5 \quad 121 = t^2$$

$$\dots\dots\dots 11 \dots\dots\dots \text{seconds}$$

(3)

(Total 7 marks)

6. In a spring, the tension (T newtons) is directly proportional to its extension (x cm). When the tension is 150 newtons, the extension is 6 cm.

- (a) Find a formula for T in terms of x .

$$T = kx$$

$$150 = k \times 6$$

$$25 = k$$

$$T = 25x \dots\dots\dots$$

(3)

- (b) Calculate the tension, in newtons, when the extension is 15 cm.

$$25 \times 15$$

$$\dots\dots\dots 375 \dots\dots\dots \text{newtons}$$

(1)

- (c) Calculate the extension, in cm, when the tension is 600 newtons.

$$600 = 25x$$

$$\div 25 \quad 24 = x$$

$$\dots\dots\dots 24 \dots\dots\dots \text{cm}$$

(1)

(Total 5 marks)