

General Certificate of Education
January 2006
Advanced Level Examination



MATHEMATICS
Unit Pure Core 3

MPC3

Wednesday 25 January 2006 9.00 am to 10.30 am

For this paper you must have:

- an 8-page answer book
 - the **blue** AQA booklet of formulae and statistical tables
 - an insert for use in Question 6 (enclosed)
- You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC3.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- Fill in the boxes at the top of the insert.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 (a) Find $\frac{dy}{dx}$ when $y = \tan 3x$. (2 marks)

(b) Given that $y = \frac{3x+1}{2x+1}$, show that $\frac{dy}{dx} = \frac{1}{(2x+1)^2}$. (3 marks)

2 Use Simpson's rule with 5 ordinates (4 strips) to find an approximation to

$$\int_1^3 \frac{1}{\sqrt{1+x^3}} dx$$

giving your answer to three significant figures. (4 marks)

3 (a) (i) Given that $f(x) = x^4 + 2x$, find $f'(x)$. (1 mark)

(ii) Hence, or otherwise, find $\int \frac{2x^3 + 1}{x^4 + 2x} dx$. (2 marks)

(b) (i) Use the substitution $u = 2x + 1$ to show that

$$\int x\sqrt{2x+1} dx = \frac{1}{4} \int \left(u^{\frac{3}{2}} - u^{\frac{1}{2}} \right) du$$
 (3 marks)

(ii) Hence show that $\int_0^4 x\sqrt{2x+1} dx = 19.9$ correct to three significant figures. (4 marks)

4 It is given that $2\operatorname{cosec}^2 x = 5 - 5 \cot x$.

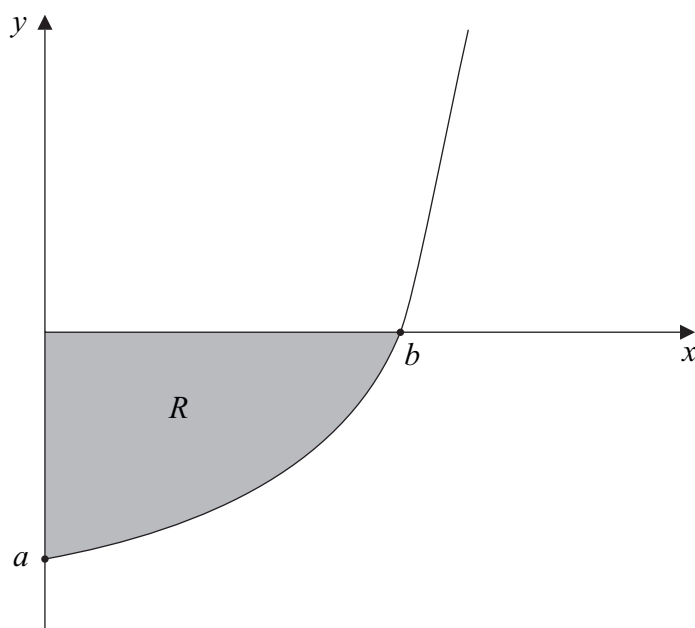
(a) Show that the equation $2\operatorname{cosec}^2 x = 5 - 5 \cot x$ can be written in the form

$$2 \cot^2 x + 5 \cot x - 3 = 0$$
 (2 marks)

(b) Hence show that $\tan x = 2$ or $\tan x = -\frac{1}{3}$. (2 marks)

(c) Hence, or otherwise, solve the equation $2\operatorname{cosec}^2 x = 5 - 5 \cot x$, giving all values of x in radians to one decimal place in the interval $-\pi < x \leq \pi$. (3 marks)

- 5 The diagram shows part of the graph of $y = e^{2x} - 9$. The graph cuts the coordinate axes at $(0, a)$ and $(b, 0)$.



- (a) State the value of a , and show that $b = \ln 3$. *(3 marks)*
- (b) Show that $y^2 = e^{4x} - 18e^{2x} + 81$. *(1 mark)*
- (c) The shaded region R is rotated through 360° about the x -axis. Find the volume of the solid formed, giving your answer in the form $\pi(p \ln 3 + q)$, where p and q are integers. *(6 marks)*
- (d) Sketch the curve with equation $y = |e^{2x} - 9|$ for $x \geq 0$. *(2 marks)*

Turn over for the next question

Turn over ►

6 [Figure 1, printed on the insert, is provided for use in this question.]

The curve $y = x^3 + 4x - 3$ intersects the x -axis at the point A where $x = \alpha$.

(a) Show that α lies between 0.5 and 1.0. (2 marks)

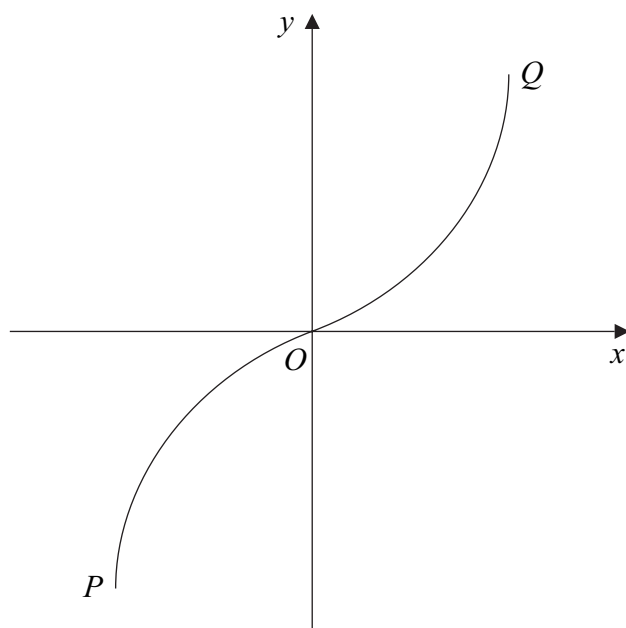
(b) Show that the equation $x^3 + 4x - 3 = 0$ can be rearranged into the form $x = \frac{3 - x^3}{4}$.
(1 mark)

(c) (i) Use the iteration $x_{n+1} = \frac{3 - x_n^3}{4}$ with $x_1 = 0.5$ to find x_3 , giving your answer to two decimal places. (3 marks)

(ii) The sketch on **Figure 1** shows parts of the graphs of $y = \frac{3 - x^3}{4}$ and $y = x$, and the position of x_1 .

On **Figure 1**, draw a cobweb or staircase diagram to show how convergence takes place, indicating the positions of x_2 and x_3 on the x -axis. (3 marks)

- 7 (a) The sketch shows the graph of $y = \sin^{-1} x$.



Write down the coordinates of the points P and Q , the end-points of the graph.

(2 marks)

- (b) Sketch the graph of $y = -\sin^{-1}(x - 1)$.

(3 marks)

- 8 The functions f and g are defined with their respective domains by

$$f(x) = x^2 \quad \text{for all real values of } x$$

$$g(x) = \frac{1}{x+2} \quad \text{for real values of } x, \quad x \neq -2$$

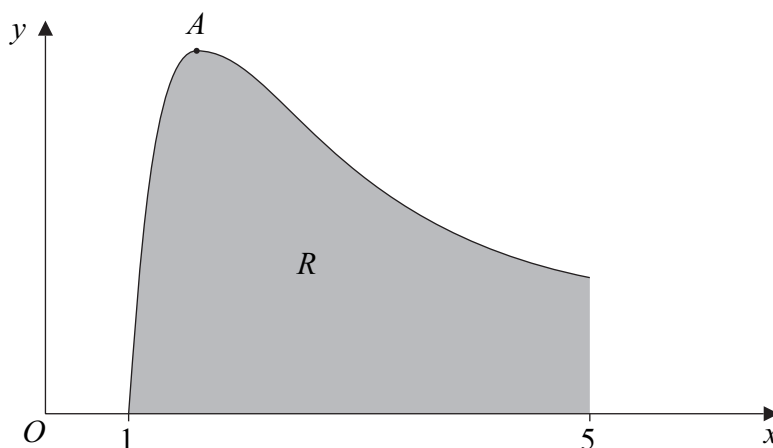
- (a) State the range of f . (1 mark)
- (b) (i) Find $fg(x)$. (1 mark)
- (ii) Solve the equation $fg(x) = 4$. (4 marks)
- (c) (i) Explain why the function f does **not** have an inverse. (1 mark)
- (ii) The inverse of g is g^{-1} . Find $g^{-1}(x)$. (3 marks)

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9 (a) Given that $y = x^{-2} \ln x$, show that $\frac{dy}{dx} = \frac{1 - 2 \ln x}{x^3}$. (4 marks)

(b) Using integration by parts, find $\int x^{-2} \ln x \, dx$. (4 marks)

(c) The sketch shows the graph of $y = x^{-2} \ln x$.



(i) Using the answer to part (a), find, in terms of e , the x -coordinate of the stationary point A . (2 marks)

(ii) The region R is bounded by the curve, the x -axis and the line $x = 5$. Using your answer to part (b), show that the area of R is

$$\frac{1}{5}(4 - \ln 5) \quad (3 \text{ marks})$$

END OF QUESTIONS

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Surname						Other Names					
Centre Number						Candidate Number					
Candidate Signature											

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Insert

Wednesday 25 January 2006 9.00 am to 10.30 am

Insert for use in **Question 6**.

Fill in the boxes at the top of this page.

Attach this insert securely to your answer book.

Turn over for Figure 1

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Figure 1 (for Question 6)