

General Certificate of Education  
January 2006  
Advanced Subsidiary Examination



**MATHEMATICS**  
**Unit Pure Core 2**

**MPC2**

Tuesday 10 January 2006 1.30 pm to 3.00 pm

**For this paper you must have:**

- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

**Instructions**

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC2.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.

**Information**

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

**Advice**

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

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Answer **all** questions.

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1 Given that  $y = 16x + x^{-1}$ , find the two values of  $x$  for which  $\frac{dy}{dx} = 0$ . (5 marks)

2 (a) Use the trapezium rule with five ordinates (four strips) to find an approximate value for

$$\int_0^4 \frac{1}{x^2 + 1} dx$$

giving your answer to four significant figures. (4 marks)

(b) State how you could obtain a better approximation to the value of the integral using the trapezium rule. (1 mark)

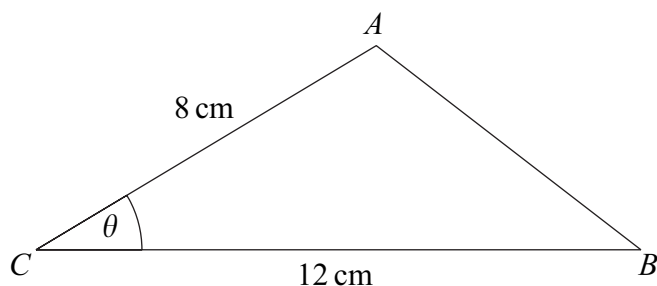
3 (a) Use logarithms to solve the equation  $0.8^x = 0.05$ , giving your answer to three decimal places. (3 marks)

(b) An infinite geometric series has common ratio  $r$ . The sum to infinity of the series is five times the first term of the series.

(i) Show that  $r = 0.8$ . (3 marks)

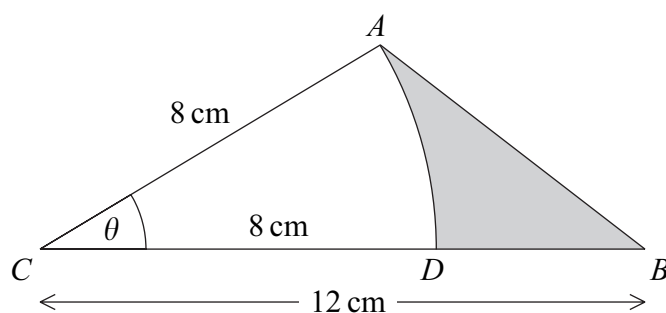
(ii) Given that the first term of the series is 20, find the least value of  $n$  such that the  $n$ th term of the series is less than 1. (3 marks)

- 4 The triangle  $ABC$ , shown in the diagram, is such that  $AC = 8$  cm,  $CB = 12$  cm and angle  $ACB = \theta$  radians.



The area of triangle  $ABC = 20$  cm<sup>2</sup>.

- (a) Show that  $\theta = 0.430$  correct to three significant figures. (3 marks)
- (b) Use the cosine rule to calculate the length of  $AB$ , giving your answer to two significant figures. (3 marks)
- (c) The point  $D$  lies on  $CB$  such that  $AD$  is an arc of a circle centre  $C$  and radius 8 cm. The region bounded by the arc  $AD$  and the straight lines  $DB$  and  $AB$  is shaded in the diagram.



Calculate, to two significant figures:

- (i) the length of the arc  $AD$ ; (2 marks)
- (ii) the area of the shaded region. (3 marks)

Turn over ►

5 The  $n$ th term of a sequence is  $u_n$ .

The sequence is defined by

$$u_{n+1} = pu_n + q$$

where  $p$  and  $q$  are constants.

The first three terms of the sequence are given by

$$u_1 = 200 \quad u_2 = 150 \quad u_3 = 120$$

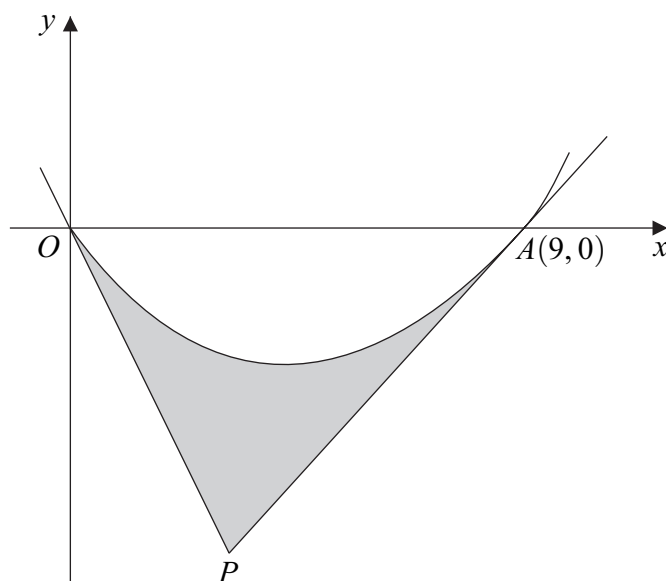
- (a) Show that  $p = 0.6$  and find the value of  $q$ . (5 marks)
- (b) Find the value of  $u_4$ . (1 mark)
- (c) The limit of  $u_n$  as  $n$  tends to infinity is  $L$ . Write down an equation for  $L$  and hence find the value of  $L$ . (3 marks)
- 6 (a) Describe the geometrical transformation that maps the curve with equation  $y = \sin x$  onto the curve with equation:
- (i)  $y = 2 \sin x$ ; (2 marks)
- (ii)  $y = -\sin x$ ; (2 marks)
- (iii)  $y = \sin(x - 30^\circ)$ . (2 marks)
- (b) Solve the equation  $\sin(\theta - 30^\circ) = 0.7$ , giving your answers to the nearest  $0.1^\circ$  in the interval  $0^\circ \leq \theta \leq 360^\circ$ . (3 marks)
- (c) Prove that  $(\cos x + \sin x)^2 + (\cos x - \sin x)^2 = 2$ . (4 marks)

7 It is given that  $n$  satisfies the equation

$$2 \log_a n - \log_a(5n - 24) = \log_a 4$$

- (a) Show that  $n^2 - 20n + 96 = 0$ . (3 marks)
- (b) Hence find the possible values of  $n$ . (2 marks)

- 8 A curve, drawn from the origin  $O$ , crosses the  $x$ -axis at the point  $A(9, 0)$ . Tangents to the curve at  $O$  and  $A$  meet at the point  $P$ , as shown in the diagram.



The curve, defined for  $x \geq 0$ , has equation

$$y = x^{\frac{3}{2}} - 3x$$

- (a) Find  $\frac{dy}{dx}$ . (2 marks)
- (b) (i) Find the value of  $\frac{dy}{dx}$  at the point  $O$  and hence write down an equation of the tangent at  $O$ . (2 marks)
- (ii) Show that the equation of the tangent at  $A(9, 0)$  is  $2y = 3x - 27$ . (3 marks)
- (iii) Hence find the coordinates of the point  $P$  where the two tangents meet. (3 marks)
- (c) Find  $\int \left( x^{\frac{3}{2}} - 3x \right) dx$ . (3 marks)
- (d) Calculate the area of the shaded region bounded by the curve and the tangents  $OP$  and  $AP$ . (5 marks)

**END OF QUESTIONS**

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