

**MATHEMATICS**  
**Unit Pure Core 1**

**MPC1**

Wednesday 20 May 2009 1.30 pm to 3.00 pm

**For this paper you must have:**

- an 8-page answer book
  - the blue AQA booklet of formulae and statistical tables.
- You must **not** use a calculator.



Time allowed: 1 hour 30 minutes

**Instructions**

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The use of calculators (scientific and graphics) is **not** permitted.

**Information**

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

**Advice**

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

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Answer **all** questions.

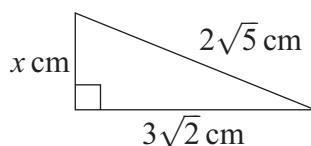
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1 The line  $AB$  has equation  $3x + 5y = 11$ .

- (a) (i) Find the gradient of  $AB$ . (2 marks)
- (ii) The point  $A$  has coordinates  $(2, 1)$ . Find an equation of the line which passes through the point  $A$  and which is perpendicular to  $AB$ . (3 marks)
- (b) The line  $AB$  intersects the line with equation  $2x + 3y = 8$  at the point  $C$ . Find the coordinates of  $C$ . (3 marks)

2 (a) Express  $\frac{5 + \sqrt{7}}{3 - \sqrt{7}}$  in the form  $m + n\sqrt{7}$ , where  $m$  and  $n$  are integers. (4 marks)

(b) The diagram shows a right-angled triangle.



The hypotenuse has length  $2\sqrt{5}$  cm. The other two sides have lengths  $3\sqrt{2}$  cm and  $x$  cm. Find the value of  $x$ . (3 marks)

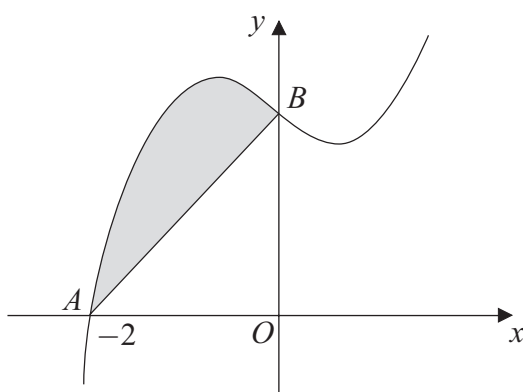
3 The curve with equation  $y = x^5 + 20x^2 - 8$  passes through the point  $P$ , where  $x = -2$ .

- (a) Find  $\frac{dy}{dx}$ . (3 marks)
- (b) Verify that the point  $P$  is a stationary point of the curve. (2 marks)
- (c) (i) Find the value of  $\frac{d^2y}{dx^2}$  at the point  $P$ . (3 marks)
- (ii) Hence, or otherwise, determine whether  $P$  is a maximum point or a minimum point. (1 mark)
- (d) Find an equation of the tangent to the curve at the point where  $x = 1$ . (4 marks)

4 (a) The polynomial  $p(x)$  is given by  $p(x) = x^3 - x + 6$ .

- (i) Find the remainder when  $p(x)$  is divided by  $x - 3$ . (2 marks)
- (ii) Use the Factor Theorem to show that  $x + 2$  is a factor of  $p(x)$ . (2 marks)
- (iii) Express  $p(x) = x^3 - x + 6$  in the form  $(x + 2)(x^2 + bx + c)$ , where  $b$  and  $c$  are integers. (2 marks)
- (iv) The equation  $p(x) = 0$  has one root equal to  $-2$ . Show that the equation has no other real roots. (2 marks)

(b) The curve with equation  $y = x^3 - x + 6$  is sketched below.



The curve cuts the  $x$ -axis at the point  $A(-2, 0)$  and the  $y$ -axis at the point  $B$ .

- (i) State the  $y$ -coordinate of the point  $B$ . (1 mark)
- (ii) Find  $\int_{-2}^0 (x^3 - x + 6) dx$ . (5 marks)
- (iii) Hence find the area of the shaded region bounded by the curve  $y = x^3 - x + 6$  and the line  $AB$ . (3 marks)

**Turn over for the next question**

**Turn over ►**

5 A circle with centre  $C$  has equation

$$(x - 5)^2 + (y + 12)^2 = 169$$

(a) Write down:

(i) the coordinates of  $C$ ; (1 mark)

(ii) the radius of the circle. (1 mark)

(b) (i) Verify that the circle passes through the origin  $O$ . (1 mark)

(ii) Given that the circle also passes through the points  $(10, 0)$  and  $(0, p)$ , sketch the circle and find the value of  $p$ . (3 marks)

(c) The point  $A(-7, -7)$  lies on the circle.

(i) Find the gradient of  $AC$ . (2 marks)

(ii) Hence find an equation of the tangent to the circle at the point  $A$ , giving your answer in the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers. (3 marks)

6 (a) (i) Express  $x^2 - 8x + 17$  in the form  $(x - p)^2 + q$ , where  $p$  and  $q$  are integers. (2 marks)

(ii) Hence write down the minimum value of  $x^2 - 8x + 17$ . (1 mark)

(iii) State the value of  $x$  for which the minimum value of  $x^2 - 8x + 17$  occurs. (1 mark)

(b) The point  $A$  has coordinates  $(5, 4)$  and the point  $B$  has coordinates  $(x, 7 - x)$ .

(i) Expand  $(x - 5)^2$ . (1 mark)

(ii) Show that  $AB^2 = 2(x^2 - 8x + 17)$ . (3 marks)

(iii) Use your results from part (a) to find the minimum value of the distance  $AB$  as  $x$  varies. (2 marks)

7 The curve  $C$  has equation  $y = k(x^2 + 3)$ , where  $k$  is a constant.

The line  $L$  has equation  $y = 2x + 2$ .

- (a) Show that the  $x$ -coordinates of any points of intersection of the curve  $C$  with the line  $L$  satisfy the equation

$$kx^2 - 2x + 3k - 2 = 0 \quad (1 \text{ mark})$$

- (b) The curve  $C$  and the line  $L$  intersect in two distinct points.

- (i) Show that

$$3k^2 - 2k - 1 < 0 \quad (4 \text{ marks})$$

- (ii) Hence find the possible values of  $k$ . (4 marks)

**END OF QUESTIONS**

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