Version : 1.0 0609

General Certificate of Education

Mathematics 6360

MPC1 Pure Core 1

Mark Scheme

2009 examination - June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Set and published by the Assessment and Qualifications Alliance.

The Assessment and Qualifications Alliance (AQA) is a company limited by guarantee registered in England and Wales (company number 3644723) and a registered charity (registered charity number 1073334).
Registered address: AQA, Devas Street, Manchester M15 6EX *Dr Michael Cresswell Director General*

Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
–*x* EE	deduct *x* marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MPC1

Q	Solution	Marks	Total	Comments
1(a)(i)	$y = -\frac{3}{5}x + \frac{11}{5}$ Or correct expression for gradient using two correct points	M1		Attempt at $y = f(x)$ **Or** answer $= \frac{3}{5}$ or $-\frac{3}{5}x$ gets M1 But answer of $\frac{3}{5}x$ gets M0
	(Gradient of AB =) $-\frac{3}{5}$	A1	2	Correct answer scores 2 marks . Condone error in rearranging formula if answer for gradient is correct.
(ii)	$m_1 m_2 = -1$	M1		Used or stated
	Gradient of perpendicular = $\frac{5}{3}$	A1√		ft their answer from (a)(i) or correct
	$y - 1 = \frac{5}{3}(x - 2)$ OE	A1	3	$5x - 3y = 7$; or $y = \frac{5}{3}x + c, \quad c = -\frac{7}{3}$ etc CSO
(b)	Eliminating x or y but must use $3x + 5y = 11$ & $2x + 3y = 8$	M1		An equation in x only or y only
	$x = 7$	A1		
	$y = -2$	A1	3	Answer only of $(7, -2)$ scores 3 marks
	Total		**8**	
2(a)	$\frac{5+\sqrt{7}}{3-\sqrt{7}} \times \frac{3+\sqrt{7}}{3+\sqrt{7}}$	M1		
	Numerator = $15 + 5\sqrt{7} + 3\sqrt{7} + 7$	m1		Condone one error or omission
	Denominator = $9 - 7$ $(= 2)$	B1		Must be seen as the denominator
	(Answer =) $11 + 4\sqrt{7}$	A1	4	
(b)	$\left(2\sqrt{5}\right)^2 = 20$ *or* $\left(3\sqrt{2}\right)^2 = 18$	B1		Either correct
	their $\left(2\sqrt{5}\right)^2 - \left(3\sqrt{2}\right)^2$ $(x^2 = 20 - 18)$	M1		Condone missing brackets and x^2 $x^2 = 2 \Rightarrow$ Bl, M1
	$(\Rightarrow x =) \sqrt{2}$	A1	3	$\pm\sqrt{2}$ scores A0 Answer only of 2 scores B0, M0 Answer only of $\sqrt{2}$ scores 3 marks
	Total		**7**	

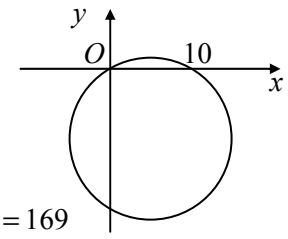
MPC1 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$\frac{dy}{dx} = 5x^4 + 40x$	M1 A1 A1	3	One of these powers correct One of these terms correct All correct (no + c etc)
(b)	$x = -2 \quad \frac{dy}{dx} = 5\times(-2)^4 + (40\times -2)$ $\frac{dy}{dx} = 5\times 16 + (40\times -2) = 0$ $\Rightarrow P$ is stationary point	M1 A1		Substitute $x = -2$ into their $\frac{dy}{dx}$ CSO Shown = 0 plus statement eg "st pt", "as required", "grad = 0"etc
	Or *their* $\frac{dy}{dx} = 0 \Rightarrow x^n = k$ $x^3 = -8 \Rightarrow x = -2$	(M1) (A1)	2	 CSO $x = 0$ need not be considered
(c)(i)	$\frac{d^2y}{dx^2} = 20x^3 + 40$ $= 20\times(-2)^3 + 40$ $(= -160 + 40) \quad = -120$	B1√ M1 A1	 3	Correct ft their $\frac{dy}{dx}$ Subst $x = -2$ into their second derivative CSO
(ii)	Maximum (value) their c(i) answer must be < 0 Other valid methods acceptable provided "maximum" is the conclusion	E1√	1	Accept minimum if their $c(i)$ *answer* > 0 and correctly interpreted Parts (i) and (ii) may be combined by candidate but –120 must be seen to award A1 in part (c)(i)
(d)	(When $x = 1$) $y = 13$ When $x = 1$, $\frac{dy}{dx} = 5 + 40$ $y =$ (their 45)$x + k$ OE Tangent has equation $y - 13 = 45(x - 1)$	B1 M1 m1 A1	 4	 Sub $x = 1$ into their $\frac{dy}{dx}$ ft their $\frac{dy}{dx}$ CSO OE $y = 45x + c, \quad c = -32$
	Total		**13**	

MPC1 (cont)

Q	Solution	Marks	Total	Comments
4(a)(i)	$p(3) = 27 - 3 + 6$ (Remainder) = 30 **Or** long division up to remainder Quotient= $x^2 + 3x + 8$ and remainder = 30 clearly stated or indicated	M1 A1 (M1) (A1)	2	p(3) attempted
(ii)	$p(-2) = -8 + 2 + 6$ $p(-2) = 0 \Rightarrow x + 2$ is factor Minimum statement required "factor"	M1 A1	2	p(–2) attempted **: NOT** long division Shown = 0 plus statement May make statement *first* such as "x+2 is a factor if p(–2) = 0"
(iii)	$b = -2$ $c = 3$ **or** long division/comparing coefficients $p(x) = (x+2)(x^2 - 2x + 3)$	B1 B1 (M1) (A1)	2	No working required for B1 + B1 Try to mark first using B marks Award M1 if B0 earned and a clear method is used Must write final answer in this form if long division has been used to get A1
(iv)	$b^2 - 4ac = (-2)^2 - 4 \times 3$ $b^2 - 4ac = -8$ (or < 0) $\Rightarrow$ no (other) real roots **Or** $(x-1)^2 + 2$ $(x-1)^2 + 2 > 0$ therefore no real roots Or $(x-1)^2 = -2$ has no real roots	M1 A1 (M1) (A1)	2	Discriminant correct from their quadratic M0 if $b = -1$, $c = 6$ used (using cubic eqn) CSO All values must be correct plus statement Completion of square for their quadratic Shown to be positive plus statement regarding no real roots
(b)(i)	$(y_B =)\ 6$	B1	1	Condone (0, 6)
(ii)	$\frac{x^4}{4} - \frac{x^2}{2} + 6x$ $\left[\ \right]_{-2}^{0} = 0 - (4 - 2 - 12)$ $= 10$	M1 A1 A1 m1 A1	5	One term correct Another term correct All correct (ignore + c or limits) $F(-2)$ attempted CSO Clearly from F(0) – F(–2)
(iii)	Area of $\Delta = \frac{1}{2} \times 2 \times 6$ $= 6$ Shaded region area = 10 – 6 = 4	M1 A1 A1	3	Condone – 2 and ft their y_B value **Or** $\int_{-2}^{0} (3x + 6)\mathrm{d}x$ and attempt to integrate Must be positive allow –6 converted to +6 CSO 10 must come from correct working
	Total		**17**	

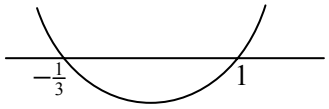
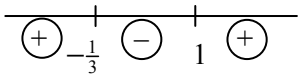
MPC1 (cont)

Q	Solution	Marks	Total	Comments
5(a)(i)	$C\,(5,-12)$	B1	1	
(ii)	Radius = 13 (or $\sqrt{169}$)	B1	1	$\pm\sqrt{169}$ or ± 13 as final answer scores B0
(b)(i)	$(-5)^2+12^2$ *or* $25+144$ $= 169 \Rightarrow$ circle passes through O	B1	1	Correct arithmetic plus statement Eg "O lies on circle", "as required" etc
(ii)	Sketch 	B1		Freehand circle through origin and cutting positive x-axis with centre in 4th quadrant Condone value 10 missing or incorrect
	$25+(p+12)^2=169$	M1		Or doubling their y_C-coordinate
	$(p+12)=\pm 12 \qquad p=-24$	A1	3	Condone use of y instead of p SC B2 for correct value of p stated or marked on diagram
(c)(i)	grad $AC = \dfrac{-12+7}{5+7}$	M1		correct expression, but ft their C
	$= -\dfrac{5}{12}$	A1	2	Condone $\dfrac{5}{-12}$
(ii)	grad tangent $= \dfrac{12}{5}$	B1 √		$\dfrac{-1}{\text{their grad } AC}$
	$y+7=\dfrac{12}{5}(x+7)$	M1		ft "their $\dfrac{12}{5}$" must be **tangent** and not AC
	$\Rightarrow 12x-5y+49=0$	A1	3	OE with **integer** coefficients with all terms on one side of the equation
	Total		**11**	

MPC1 (cont)

Q	Solution	Marks	Total	Comments
6(a)(i)	$(x-4)^2$ *or* $p=4$	B1		ISW for $p=-4$ if $(x-4)^2$ seen
	$+1$ *or* $q=1$	B1	2	
(ii)	(Minimum value is) 1	B1√	1	Correct or FT "their q" (NOT coords)
(iii)	(Minimum occurs when $x=$)4	B1√	1	Correct or FT "their p" – may use calculus Condone (p, **) for this B1 mark
(b)(i)	$(x-5)^2 = x^2-10x+25$	B1	1	
(ii)	$(x-5)^2+(7-x-4)^2$	M1		Condone one slip in one bracket May be seen under $\sqrt{}$ sign
	$=(x-5)^2+(3-x)^2$			
	$=x^2-10x+25+9-6x+x^2$	A1		From a fully correct expression
	$AB^2=2x^2-16x+34$			
	$=2(x^2-8x+17)$	A1	3	AG CSO
(iii)	Minimum $AB^2=2\times$ "their (a)(ii)"	M1		Or use of their $x=4$ in expression Or use of their $B(4, 3)$ and $A(5, 4)$ in distance formula M0 if calculus used Answer only of $2\times$ "their (a)(ii)" scores M1, A0
	Minimum $AB=\sqrt{2}$	A1	2	
	Total		**10**	

MPC1 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$k\left(x^2+3\right)=2x+2$			
	$\Rightarrow kx^2-2x+3k-2=0$	B1	1	AG OE all terms on one side and = 0
(b)(i)	Discriminant = $(-2)^2-4k(3k-2)$	M1		Condone one slip (including x is one slip) Condone 2^2 or 4 as first term
	$=4-12k^2+8k$	A1		condone recovery from missing brackets
	Two distinct real roots $\Rightarrow b^2-4ac>0$	B1√		"their discriminant in terms of k" >0
	$4-12k^2+8k>0$			Not simply the statement $b^2-4ac>0$
	$\Rightarrow 12k^2-8k-4<0$			Change from > 0 to < 0 and divide by 4
	$\Rightarrow 3k^2-2k-1<0$	A1	4	AG CSO
(ii)	$(3k+1)(k-1)$	M1		**Correct** factors or **correct** use of formula
	Critical values 1 and $-\frac{1}{3}$	A1		May score M1, A1 for correct critical values seen as part of incorrect final answer with or without working
	Use of sign diagram or sketch	M1		If previous A1 earned, sign diagram or sketch must be correct for M1
				
				Otherwise, M1 may be earned for an attempt at the sketch or sign diagram using their critical values.
	$\Rightarrow -\frac{1}{3}<k<1$ or $1>k>-\frac{1}{3}$	A1	4	Full marks for correct final answer with or without working $\leqslant$ loses final A mark
	condone $-\frac{1}{3}<k$ AND $k<1$ for full marks but not OR or "," instead of AND			*Answer only* of $1<k<-\frac{1}{3}$ or $k<-\frac{1}{3};k<1$ *etc* scores M1,A1,M0 since the correct critical values are evident *Answer only* of $\frac{1}{3}<k<1$ etc where critical values are not both correct gets M0,M0
	Total		**9**	
	TOTAL		**75**	