

General Certificate of Education
January 2006
Advanced Subsidiary Examination



MATHEMATICS
Unit Pure Core 1

MPC1

Tuesday 10 January 2006 1.30 pm to 3.00 pm

For this paper you must have:

- an 8-page answer book
 - the **blue** AQA booklet of formulae and statistical tables
- You must **not** use a calculator.



Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC1.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The use of calculators (scientific and graphics) is **not** permitted.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 (a) Simplify $(\sqrt{5} + 2)(\sqrt{5} - 2)$. (2 marks)

(b) Express $\sqrt{8} + \sqrt{18}$ in the form $n\sqrt{2}$, where n is an integer. (2 marks)

2 The point A has coordinates $(1, 1)$ and the point B has coordinates $(5, k)$.

The line AB has equation $3x + 4y = 7$.

(a) (i) Show that $k = -2$. (1 mark)

(ii) Hence find the coordinates of the mid-point of AB . (2 marks)

(b) Find the gradient of AB . (2 marks)

(c) The line AC is perpendicular to the line AB .

(i) Find the gradient of AC . (2 marks)

(ii) Hence find an equation of the line AC . (1 mark)

(iii) Given that the point C lies on the x -axis, find its x -coordinate. (2 marks)

3 (a) (i) Express $x^2 - 4x + 9$ in the form $(x - p)^2 + q$, where p and q are integers. (2 marks)

(ii) Hence, or otherwise, state the coordinates of the minimum point of the curve with equation $y = x^2 - 4x + 9$. (2 marks)

(b) The line L has equation $y + 2x = 12$ and the curve C has equation $y = x^2 - 4x + 9$.

(i) Show that the x -coordinates of the points of intersection of L and C satisfy the equation

$$x^2 - 2x - 3 = 0 \quad (1 \text{ mark})$$

(ii) Hence find the coordinates of the points of intersection of L and C . (4 marks)

4 The quadratic equation $x^2 + (m + 4)x + (4m + 1) = 0$, where m is a constant, has equal roots.

(a) Show that $m^2 - 8m + 12 = 0$. (3 marks)

(b) Hence find the possible values of m . (2 marks)

5 A circle with centre C has equation $x^2 + y^2 - 8x + 6y = 11$.

(a) By completing the square, express this equation in the form

$$(x - a)^2 + (y - b)^2 = r^2 \quad (3 \text{ marks})$$

(b) Write down:

(i) the coordinates of C ; (1 mark)

(ii) the radius of the circle. (1 mark)

(c) The point O has coordinates $(0, 0)$.

(i) Find the length of CO . (2 marks)

(ii) Hence determine whether the point O lies inside or outside the circle, giving a reason for your answer. (2 marks)

6 The polynomial $p(x)$ is given by

$$p(x) = x^3 + x^2 - 10x + 8$$

(a) (i) Using the factor theorem, show that $x - 2$ is a factor of $p(x)$. (2 marks)

(ii) Hence express $p(x)$ as the product of three linear factors. (3 marks)

(b) Sketch the curve with equation $y = x^3 + x^2 - 10x + 8$, showing the coordinates of the points where the curve cuts the axes.

(You are not required to calculate the coordinates of the stationary points.) (4 marks)

7 The volume, $V \text{ m}^3$, of water in a tank at time t seconds is given by

$$V = \frac{1}{3}t^6 - 2t^4 + 3t^2, \quad \text{for } t \geq 0$$

(a) Find:

(i) $\frac{dV}{dt}$; *(3 marks)*

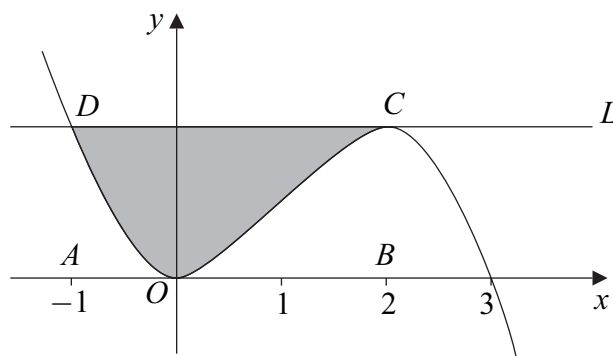
(ii) $\frac{d^2V}{dt^2}$. *(2 marks)*

(b) Find the rate of change of the volume of water in the tank, in $\text{m}^3 \text{ s}^{-1}$, when $t = 2$. *(2 marks)*

(c) (i) Verify that V has a stationary value when $t = 1$. *(2 marks)*

(ii) Determine whether this is a maximum or minimum value. *(2 marks)*

- 8 The diagram shows the curve with equation $y = 3x^2 - x^3$ and the line L .



The points A and B have coordinates $(-1, 0)$ and $(2, 0)$ respectively. The curve touches the x -axis at the origin O and crosses the x -axis at the point $(3, 0)$. The line L cuts the curve at the point D where $x = -1$ and touches the curve at C where $x = 2$.

- (a) Find the area of the rectangle $ABCD$. (2 marks)
- (b) (i) Find $\int (3x^2 - x^3) dx$. (3 marks)
- (ii) Hence find the area of the shaded region bounded by the curve and the line L . (4 marks)
- (c) For the curve above with equation $y = 3x^2 - x^3$:
- (i) find $\frac{dy}{dx}$; (2 marks)
- (ii) hence find an equation of the tangent at the point on the curve where $x = 1$; (3 marks)
- (iii) show that y is decreasing when $x^2 - 2x > 0$. (2 marks)
- (d) Solve the inequality $x^2 - 2x > 0$. (2 marks)

END OF QUESTIONS

There are no questions printed on this page

There are no questions printed on this page

There are no questions printed on this page