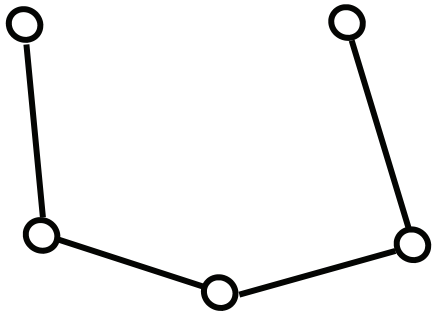
Year 12 Mathematics Challenge 2010—Round 2
Graph Colouring and Chromatic Polynomials

Question 1:

How many ways can this path be coloured using three colours?

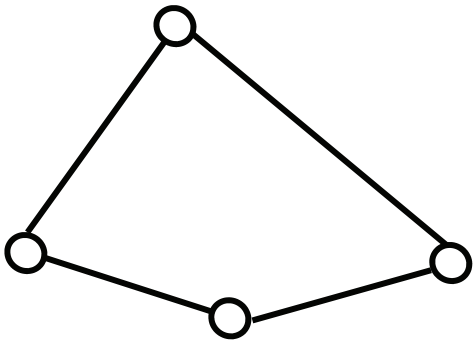


Question 2:

What is the least number of colours that can be used to colour any path?

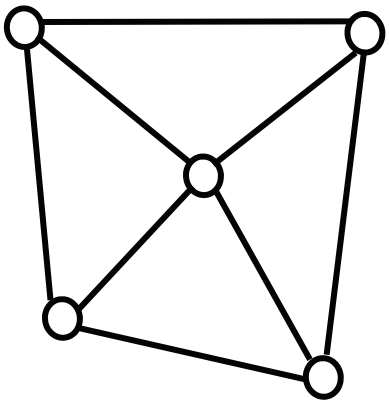
Question 3:

How many colourings are there for this cycle using three colours?



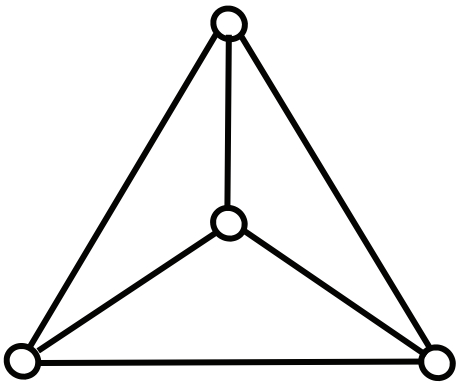
Question 4:

How many colourings are there for this graph using four colours?



Question 5

Produce a minimal vertex colouring of the dual of the following complete graph on 4 vertices.



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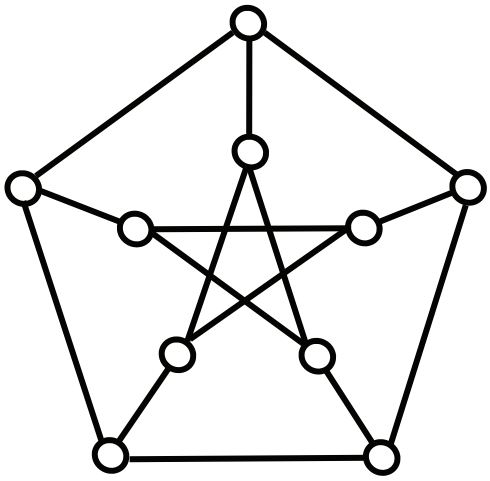
Question 6:

Produce a minimal proper colouring of the Petersen graph opposite.



Given the chromatic polynomial for the Petersen graph is:

$$t(t-1)(t-2)(t^7 - 12t^6 + 67t^5 - 230t^4 + 529t^3 - 814t^2 + 775t - 352)$$

Question 7:

How do you know that your colouring is minimal?

Question 8:

How many minimal colourings are there?

We define the edge colouring of a graph with the same rules as the vertex colouring, only we are colouring the edges instead of the vertices.

Question 9:

Given this definition, what is the minimum number of colours required for an edge colouring of the Petersen graph?

Year 12 Mathematics Challenge 2010—Round 2 Answers

School:

Question 1:

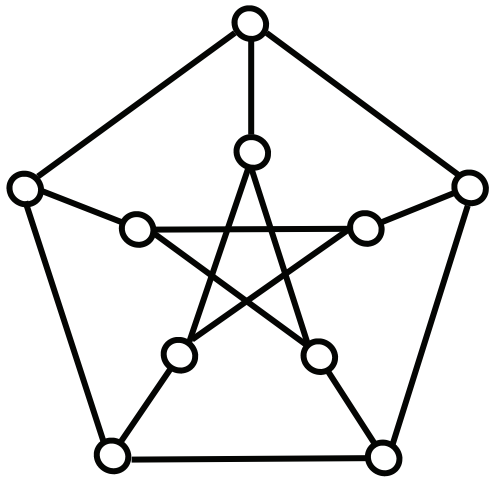
Question 2:

Question 3:

Question 4:

Question 5:

Question 6:



Question 7:

Question 8:

Question 9:

Year 12 Mathematics Challenge 2010—Round 2 Answers

Mark Sheet:
Total 30 marks

Question 1:
48 3 marks

Question 2:
2 colours 2 marks

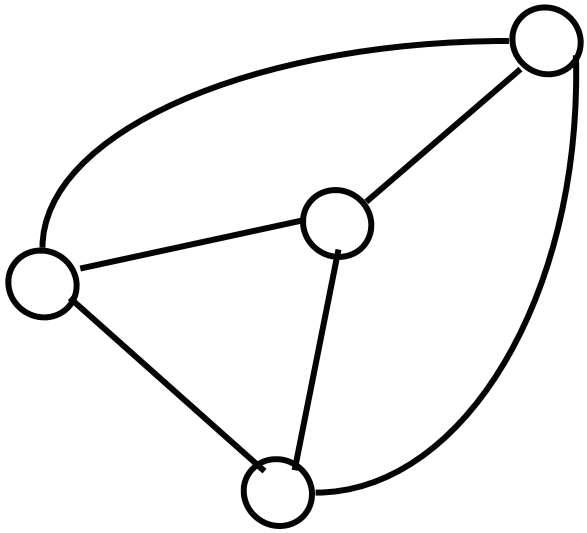
Question 3:
18 3 marks

Question 4:
72 3 marks

Question 5:

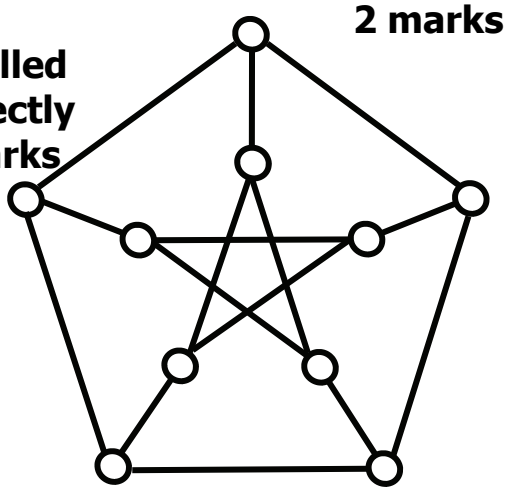
Dual correctly drawn
2 marks

4 colours labelled
2 marks



Question 6: 3 colours used
2 marks

Labelled correctly
3 marks



Question 7:

C= 0 for t = 0, 1, 2

2 marks

Hence t = 3 is the first non zero value.

2 marks

Question 8:
120 3 marks

Question 9:
4 colours 3 marks

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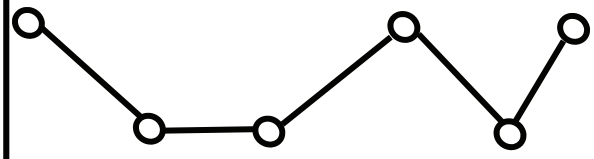
Graph Colouring and Chromatic Polynomials

C(n,t) - number of colourings where:
t - the number of colours available for the colouring
n - the number of vertices in the graph

Path

$C = t(t-1)^{n-1}$

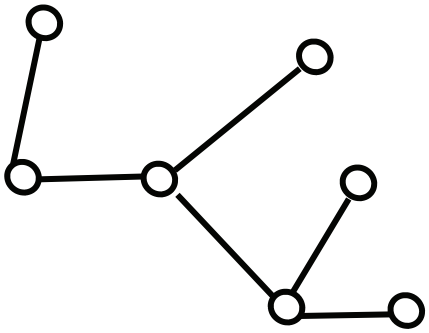


5 colours (t=5)
6 vertices (n=6)

$C = 5 \times (5-1)^{6-1}$
$= 5120$

Tree

$C = t(t-1)^{n-1}$

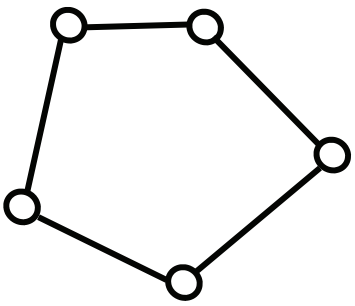


3 colours (t=3)
7 vertices (n=7)

$C = 3 \times (3-1)^{7-1}$
$= 192$

Cycle

$C = (t-1)^n + (-1)^n(t-1)$

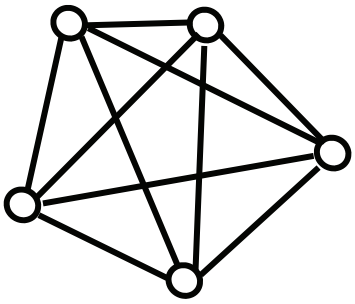


4 colours (t=4)
5 vertices (n=5)

$C = (4-1)^5 + (-1)^5(4-1)$
$= 240$

Complete Graph

$C = t(t-1)(t-2)\ldots(t-(n-1))$



6 colours (t=6)
5 vertices (n=5)

$C = 6(6-1)(6-2)(6-3)(6-2)$
$= 720$