

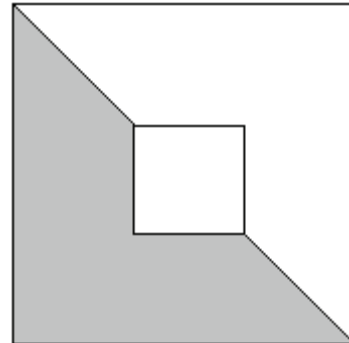
# Year 12 Mathematics Challenge 2010—Round 1

## Pair A

### Question A1:

A unit square is centred in a square with sides of length 3.

What fraction of the large square is shaded?



### Question A2:

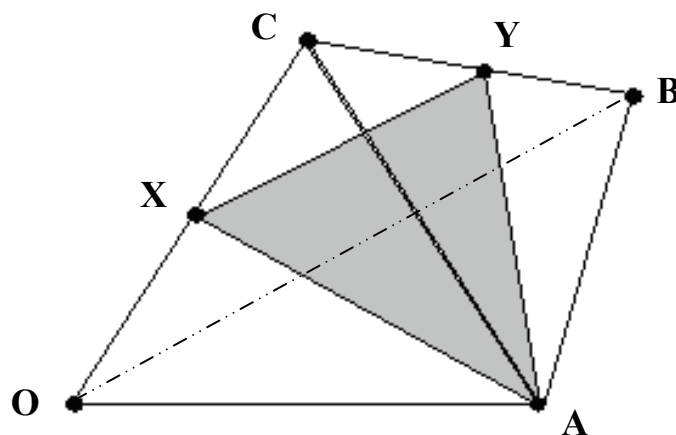
Given the following five equations of cartesian graphs:

$$x^2 + y^2 = 9 \quad xy = 5 \quad x + y = 4 \quad x^2 - y^2 = 9 \quad y = x^2 - 9$$

(i) Which curves passes closest to the origin?

(ii) Which curve does not intersect the line  $y = x$ ?

### Question A3:



A regular tetrahedron OABC has edges of length 2.  
X is the mid-point of OC and Y is the mid-point of BC.

Calculate the area of triangle XAY, giving your answer in surd form.

## **Year 12 Mathematics Challenge 2010—Round 1 Pair A**

### **Question A4:**

An astronaut lands on the equator of a spherical asteroid. He travels due north 100km, without reaching the pole, then east 100km, then south 100km. He does not pass the same point more than once and finds that he is due east of his original starting point by 200km. How many kilometres would he now need to travel by continuing his journey in an easterly direction in order to reach his original starting point?

### **Question A5:**

A postman delivers five letters, one to each of five houses. Being in a hurry, he delivers the letters so that only 2 of the houses receive the correct letter.

In how many different ways can the letters be delivered?

# Year 12 Mathematics Challenge 2010—Round 1

## Pair B

### Question B1:

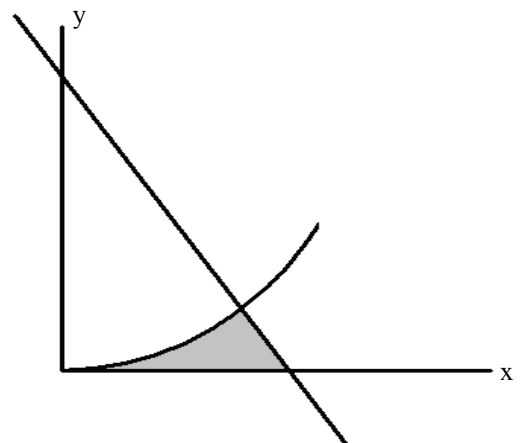
A set of cards numbered 1 to 25 are shuffled. Four cards are then dealt. What is the probability of these being dealt in ascending order?

(Give your answer as a fraction.)

### Question B2:

Given the curve  $y = \frac{3}{2}x^2$  and the line  $6x + y = 18$ ;

Calculate the area of the shaded region.



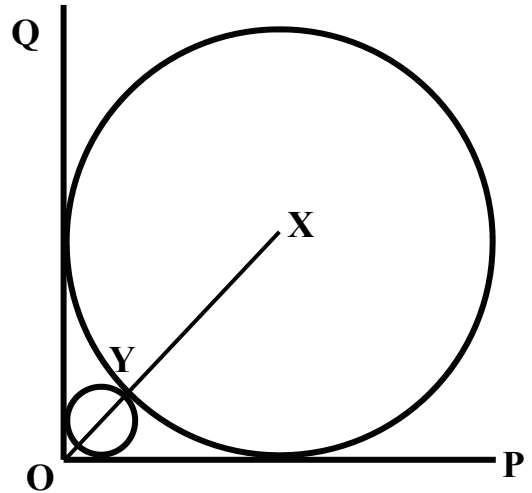
# Year 12 Mathematics Challenge 2010—Round 1 Pair B

## Question B3:

The diagram shows two circles, touching at  $Y$ , which are tangential to the perpendicular lines  $OP$  and  $OQ$ .  $X$  is the centre of the larger circle.

The small circle has a unit radius.

- (i) Calculate  $OY$
- (ii) Calculate the radius of the larger circle, giving your answers in surd form.



## Question B4:

A cube fits perfectly inside a sphere, which fits perfectly inside a cube, which fits perfectly inside a sphere, which fits perfectly inside a cube.

What is the ratio of the volume of the largest cube to the smallest cube?

## Question B5:

Find the 5 integer values of  $x$  that satisfy

$$\left(\frac{2x}{x^2 - 24}\right)^x = 1$$

**Year 12 Mathematics Challenge  
2010—Round 1 Answers**

**School:**

**Question A1:**

**Question B1:**

**Question A2:**

**Question B2:**

**Question A3:**

**Question B3:**

**Question A4:**

**Question B4:**

**Question A5:**

**Question B5:**

**Year 12 Mathematics Challenge  
2010—Round 1 Answers**

**Mark Sheet:  
4 marks each question**

**Question A1:**

**4/9                      4 marks**

**Question B1:**

**1/24                      4 marks**

**Question A2:**

**(i)  $x+y = 4$                       2 marks**

**(ii)  $x^2 - y^2 = 9$                       2 marks**

**Question B2:**

**7                              4 marks**

**Question A3:**

**$(\sqrt{11})/4$                       4 marks**

**Question B3:**

**$3 + 2\sqrt{2}$                       4 marks**

**$\frac{\sqrt{2} + 1}{\sqrt{2} - 1}$  give 3 marks**

**Question A4:**

**400km                      4 marks**

**Question B4:**

**27:1                              4 marks**

**Allow 27**

**Question A5:**

**20                              4 marks**

**Question B5:**

**$X = 6, -6, 4, -4, 0$**

**All correct 4 marks  
Minus 1 for each error**