Progression maps: Using and applying mathematics – Reasoning

Step 1

- Objective: Explain why an answer is correct.

Step 2

- Objective: Understand a general statement by finding particular examples that match it.

Step 3

- Objective: Try out ideas to find a pattern or solution.

Step 4

- Objective: Make general statements, based on evidence produced, and explain reasoning.

Step 5

- Objective: Solve problems and investigate in a range of contexts, explaining and justifying methods and conclusions; begin to generalise and to understand the significance of a counter-example.

Step 6

- Objective: Draw simple conclusions and explain reasoning; suggest extensions to problems; conjecture and generalise.

Step 7

- Objective: Use logical argument to establish the truth of a statement; begin to give mathematical justifications and test by checking particular cases.

Step 8

- Objective: Present a concise reasoned argument, using symbols, diagrams, graphs and related explanatory texts.

Step 9

- Objective: Show some insight into mathematical structure by using pattern and symmetry to justify generalisations, arguments or solutions.

Step 10

- Objective: Appreciate the difference between mathematical explanation and experimental evidence.

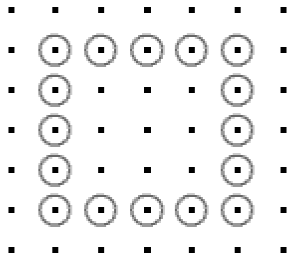
Step 1 Objective

Explain why an answer is correct.

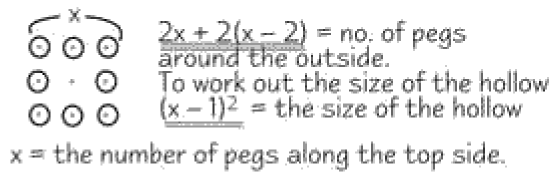
Examples of what pupils should know and be able to do

Hollow Squares

Here is a hollow square.



- How many pegs form the square on the outside?
- How many pegs are there in the hollow?
- Draw some more hollow squares.
- Investigate.



Examples drawn from **Hollow Squares**.

Pupils can draw and make some hollow squares and count the dots in the middle (or on the sides).

Probing questions

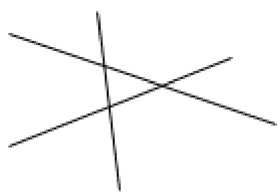
How did you work this out?

How do you know your answer is correct?

What if pupils find this a barrier?

Line Crossings

- Draw three straight lines (line segments) so that some cross over each other.
- How many crossings are there?
- Try different arrangements of the lines. What is the maximum number of possible crossings?
- Try using more lines.
- Is there a rule for the maximum for any number of lines? If so, write it down.



Use the problem **Line Crossings**:

- Draw four straight lines so some cross over each other.
- How many crossings are there?
- Show me which crossings you counted.

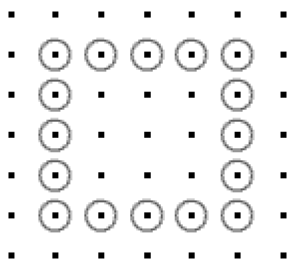
Step 2 Objective

Understand a general statement by finding particular examples that match it.

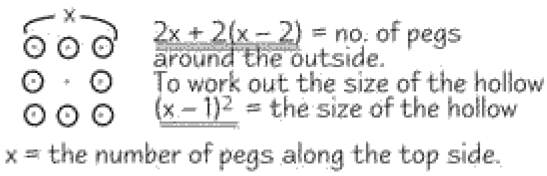
Examples of what pupils should know and be able to do

Hollow Squares

Here is a hollow square.

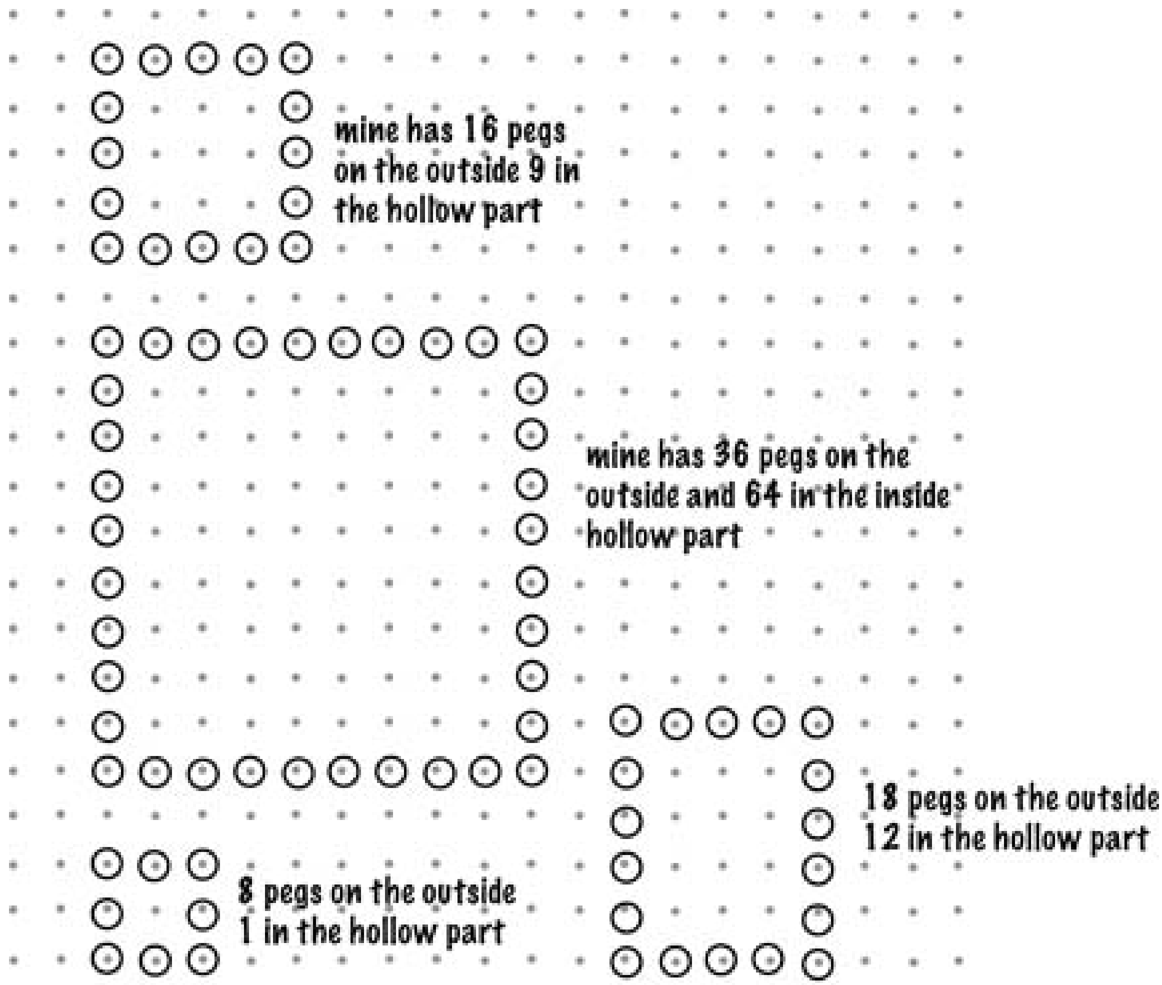


- How many pegs form the square on the outside?
- How many pegs are there in the hollow?
- Draw some more hollow squares.
- Investigate.



Examples drawn from **Hollow Squares**.

Pupils can draw and make some hollow squares and count the dots in the middle (or on the sides).



- Example 1 – 16 pegs on outside
- Example 2 – 36 pegs on outside
- Example 3 – 18 pegs on outside
- Example 4 – 8 pegs on outside

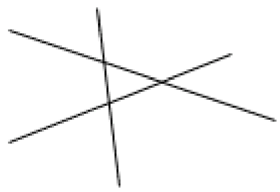
Probing questions

Can you give me some other examples that match this statement? Can you give me some examples that don't match it?

What if pupils find this a barrier?

Line Crossings

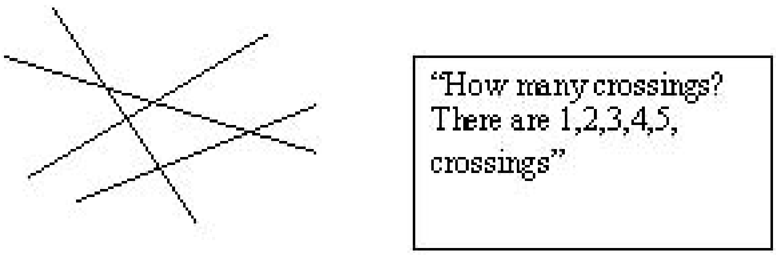
- Draw three straight lines (line segments) so that some cross over each other.
- How many crossings are there?
- Try different arrangements of the lines. What is the maximum number of possible crossings?
- Try using more lines.
- Is there a rule for the maximum for any number of lines? If so, write it down.

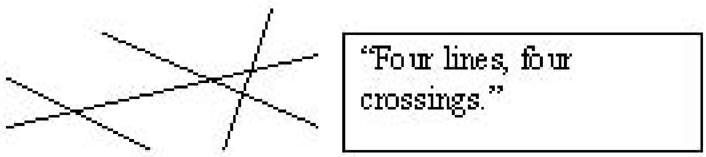


Use the problem **Line Crossings**:

- Can you draw a different arrangement?
- How many crossings are there?
- Now try another arrangement and explain how many crossings there are.
- How would you write this down?

"How many crossings? There are one, two, three, four, five crossings"





"Four lines, four crossings"

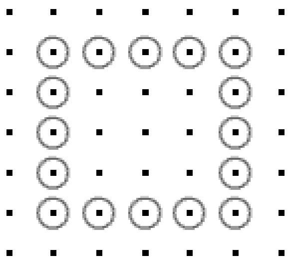
Step 3 Objective

Try out ideas to find a pattern or solution.

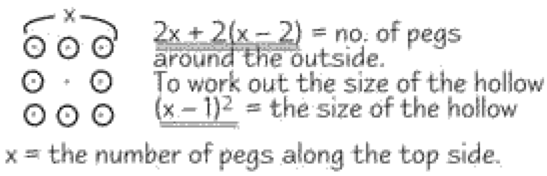
Examples of what pupils should know and be able to do

Hollow Squares

Here is a hollow square.



- How many pegs form the square on the outside?
- How many pegs are there in the hollow?
- Draw some more hollow squares.
- Investigate.



Examples drawn from **Hollow Squares**:

Pupils notice that the number of pegs on the outside is always even.

Probing questions

Have you found a pattern? What did you do that helped?

Have you found a solution? How did you do it?

What if pupils find this a barrier?

Line Crossings

- Draw three straight lines (line segments) so that some cross over each other.
- How many crossings are there?
- Try different arrangements of the lines. What is the maximum number of possible crossings?
- Try using more lines.
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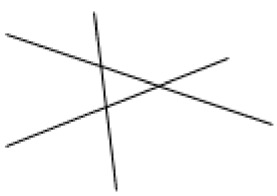


Use the problem **Line Crossings**:

- Draw some diagrams with intersections.
- What do you notice about the number of lines and the number of intersections?

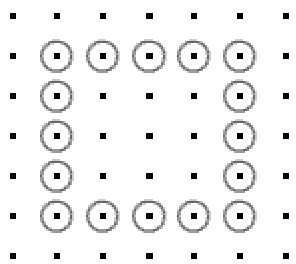
Step 4 Objective

Make general statements, based on evidence produced, and explain reasoning.

Examples of what pupils should know and be able to do

Hollow Squares

Here is a hollow square.



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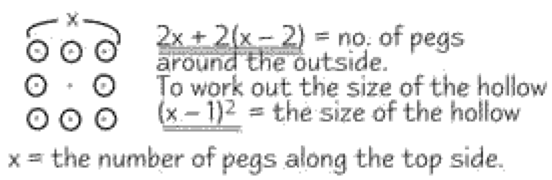


Examples drawn from **Hollow Squares**:

Pupil notices that the number of pegs on the outside is in the four times table.

"I noticed that all outside numbers belong to the four times table. In this investigation I found out that every square you draw gets bigger and there are more dots inside the squares. Also I have found out that the number of pegs in turn go odd, even and so on. I have tried many different shapes for this investigation – squares, triangles, hexagons and pentagons."

Probing questions

What have you found out?

Can you express this as a general statement that someone else could test?

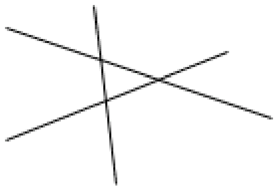
Why do you think your general statement is true?

What if pupils find this a barrier?

Line Crossings

- Draw three straight lines (line segments) so that some cross over each other.
- How many crossings are there?
- Try different arrangements of the lines. What is the maximum number of possible crossings?
- Try using more lines.

- Is there a rule for the maximum for any number of lines? If so, write it down.



Use the problem **Line Crossings**:

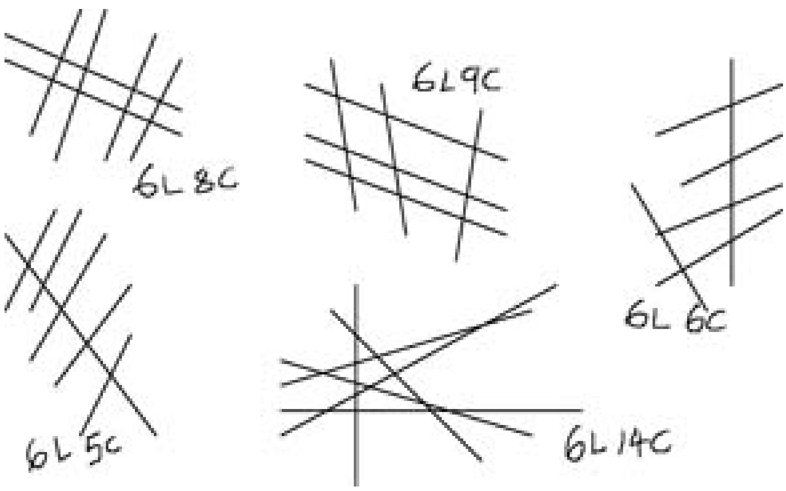
- Look at the different diagrams with five (or six) lines.
- When there are only a few crossings what can you say about the lines?
- What do the lines look like when there are a lot of crossings?
- Can you explain how you can get more (or fewer) crossings?

"With five lines I can draw a lot of patterns with different number of crossings."

"When lines are parallel there are no crossings."

"You have to be careful because if you make some lines longer they will cross other lines."

"The more jumbled the picture the more crossings there are."



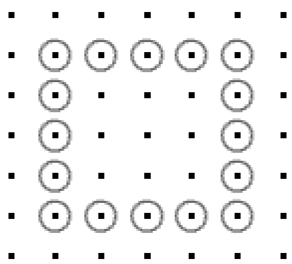
Step 5 Objective

Solve problems and investigate in a range of contexts, explaining and justifying methods and conclusions; begin to generalise and to understand the significance of a counter-example.

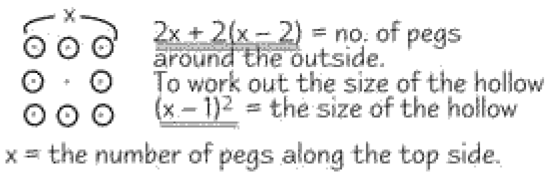
Examples of what pupils should know and be able to do

Hollow Squares

Here is a hollow square.



- How many pegs form the square on the outside?
- How many pegs are there in the hollow?
- Draw some more hollow squares.
- Investigate.



Examples drawn from **Hollow Squares**:

Pupil notices that the number of pegs on the outside increases by four as the number along each side of the square increases by one. Partially explains why the number is a multiple of four by referring to the number of sides of a square.

'First of all the dots on the outside go up in four, for example 12, 18, and 20.'

Probing questions

What have you noticed?

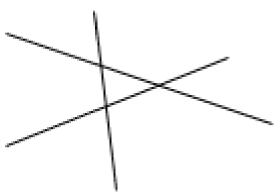
Will this always happen? How do you know?

Can you generalise from this?

What if pupils find this a barrier?

Line Crossings

- Draw three straight lines (line segments) so that some cross over each other.
- How many crossings are there?
- Try different arrangements of the lines. What is the maximum number of possible crossings?
- Try using more lines.
- Is there a rule for the maximum for any number of lines? If so, write it down.



Use the problem **Line Crossings**:

- Pupils draw different numbers of lines and crossings.
- What can you say about the number of crossings when there are a few lines?
- Explain what happens to the number of crossings when you get more lines.

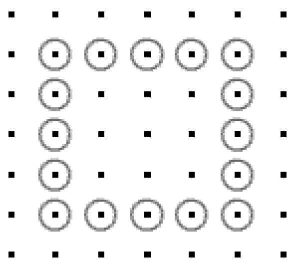
Step 6 Objective

Draw simple conclusions and explain reasoning; suggest extensions to problems; conjecture and generalise.

Examples of what pupils should know and be able to do

Hollow Squares

Here is a hollow square.



- How many pegs form the square on the outside?
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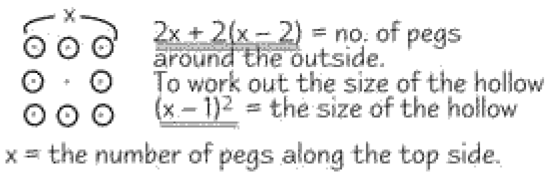
- Draw some more hollow squares.
- Investigate.



Examples drawn from **Hollow Squares**:

Pupil makes a table of results systematically with most of results correct, may notice that the number of pegs in the hollow increases by two more each time.

There are 16 pegs on the outside. The hollow is nine pegs.

Area of hollow square	Number of pegs on outside	Number of pegs in hollow
• 3 x 3 • 4 x 4 • 5 x 5 • 6 x 6 • 7 x 7 • 8 x 8 • 9 x 9 • 10 x 10	• 8 • 12 • 16 • 20 • 24 • 28 • 32 • 36	• 1 • 4 • 9 • 16 • 25 • 36 • 49 • 64

The number of pegs on the outside goes up in fours.

The number of pegs in the hollow goes up in twos.

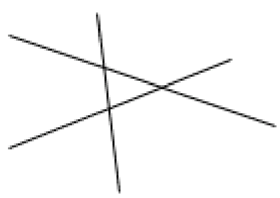
Probing questions

What have you found out? Why do you think this is? How might you take this further? What do you think might happen?

What if pupils find this a barrier?

Line Crossings

- Draw three straight lines (line segments) so that some cross over each other.
- How many crossings are there?
- Try different arrangements of the lines. What is the maximum number of possible crossings?
- Try using more lines.
- Is there a rule for the maximum for any number of lines? If so, write it down.



Use the problem **Line Crossings**:

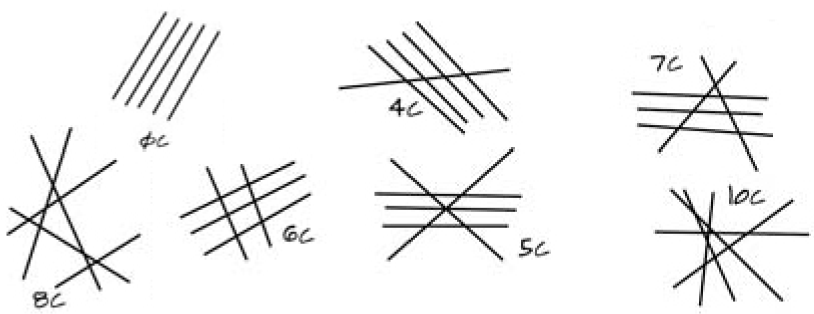
- How could you organise your work so that you go from smallest (number of intersections) to largest?
- What patterns can you see? How would you explain them?
- Why do you think this is true?

"With five lines I can draw a lot of patterns with different number of crossings."

"When lines are parallel there are no crossings."

"You have to be careful because if you make some lines longer they will cross other lines."

"The more jumbled the picture the more crossings there are."



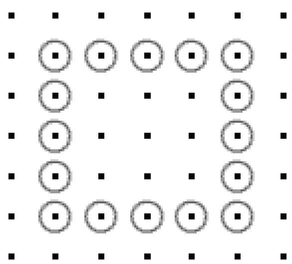
Step 7 Objective

Use logical argument to establish the truth of a statement; begin to give mathematical justifications and test by checking particular cases.

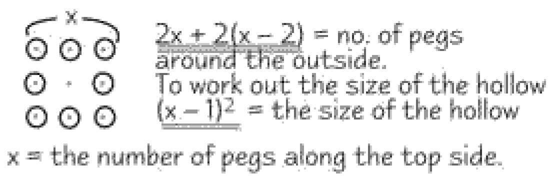
Examples of what pupils should know and be able to do

Hollow Squares

Here is a hollow square.



- How many pegs form the square on the outside?
- How many pegs are there in the hollow?
- Draw some more hollow squares.
- Investigate.



Examples drawn from **Hollow Squares**:

As in Step 6 but with mathematical explanations about the patterns in the results.

Probing questions

How would you convince a friend? How about a penpal?

How would you convince me?

Does this work for any value? How do you know?

What special cases should you check?

What if pupils find this a barrier?

Line Crossings

- Draw three straight lines (line segments) so that some cross over each other.
- How many crossings are there?
- Try different arrangements of the lines. What is the maximum number of possible crossings?
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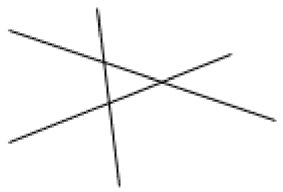


Use the problem **Line Crossings**:

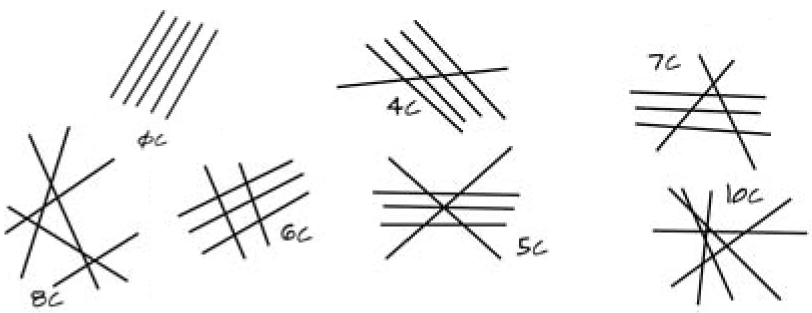
- Put all of your results (for greatest number of crossings) into a table.
- What patterns can you see? How would you explain them?
- Look at your table/pattern. How might you know if some of your results are wrong?

'With five lines I can draw a lot of patterns with different number of crossings.'

'When lines are parallel there are no crossings.'

'You have to be careful because if you make some lines longer they will cross other lines.'

The more jumbled the picture the more crossings there are.



Step 8 Objective

Present a concise reasoned argument, using symbols, diagrams, graphs and related explanatory texts.

Examples of what pupils should know and be able to do

Hollow Squares

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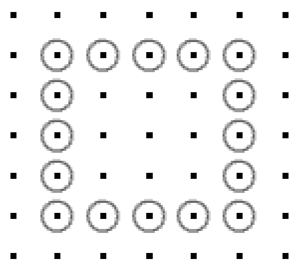

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- Draw some more hollow squares.
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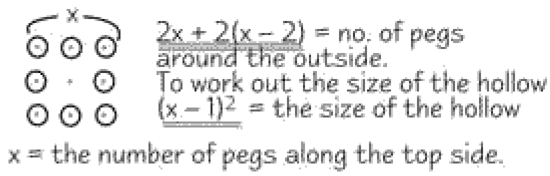


Examples drawn from **Hollow Squares**:

Pupil writes a connected chain of reasoning to explain why the number of pegs on the outside is a multiple of four.

Or

Pupil notices that the number of pegs in the hollow produces the sequence of square numbers.

Every time the number of dots around the outside increases by four.

'After making the table I discovered that the number of squares on the inside of a square was got by squaring the sides of the previous square, for example, square 1 is 2x2 square.'

Probing questions

Why did you use that graph/diagram? What does the graph/diagram reveal about the problem?

How did you decide on the scales on your graph?

Would a different scale change your conclusions?

Can you explain the pattern that is shown here? Can you explain the rule? What would you expect in the next case?

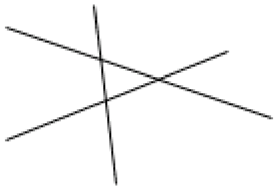
Can you explain why some of the results do not follow the same pattern?

Is your generalisation always true? How do you know? What if…?

What if pupils find this a barrier?

Line Crossings

- Draw three straight lines (line segments) so that some cross over each other.
- How many crossings are there?
- Try different arrangements of the lines. What is the maximum number of possible crossings?
- Try using more lines.
- Is there a rule for the maximum for any number of lines? If so, write it down.



Use the problem **Line Crossings**:

- Look at the table of results and explain the patterns you see.
- Can you predict what the next number in the table (pattern) would be?
- Are you sure? (Check by drawing this.)

Lines	Crossings
1	0
2	1
3	3
4	6
5	10
6	15

With more lines you get more crossings.
With each extra line the number of crossings goes up 1,2,3,4,5...
When you add another line you get more extra crossings because there are more lines to cross each time.

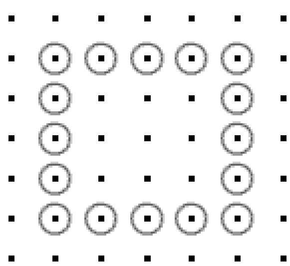
Step 9 Objective

Show some insight into mathematical structure by using pattern and symmetry to justify generalisations, arguments or solutions.

Examples of what pupils should know and be able to do

Hollow Squares

Here is a hollow square.



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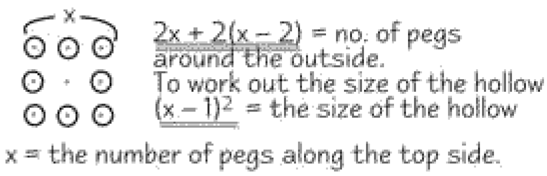

Examples drawn from **Hollow Squares**:

From results generated pupil describes in words how to work out the number of pegs inside or outside.

'The number on the outside goes up by four each time. You can work out the middle by if it was six by six it would take two off so it would be four by four. Then you would times four by four and that would give you the answer.'

Probing questions

How does your generalisation link to the original problem? Explain; for example, multiply by six, add two, squaring (vary according to context).

What if you started with … instead of …? How might your generalisation/solution be different?

What if pupils find this a barrier?

Line Crossings

- Draw three straight lines (line segments) so that some cross over each other.
- How many crossings are there?
- Try different arrangements of the lines. What is the maximum number of possible crossings?
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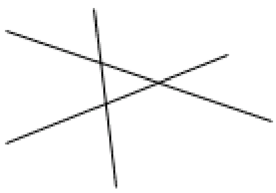


Use the problem **Line Crossings**:

- Look at the pattern as you add another line; how many extra crossings does it make?
- Why can't there be any more crossings? Can you explain why?
- If you had 12 lines and you added one more line, what would be the greatest number of possible crossings? Why?

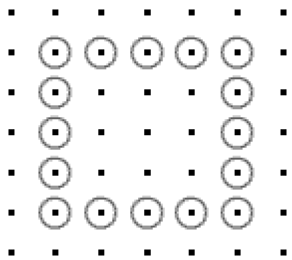
Step 10 Objective

Appreciate the difference between mathematical explanation and experimental evidence.

Examples of what pupils should know and be able to do

Hollow Squares

Here is a hollow square.



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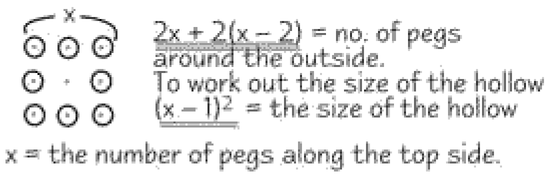

Examples drawn from **Hollow Squares**:

"I worked this out by using two formulas or I could have just carried on with the patterns mentioned above."

"If a square is 2cm by 2cm there are three pegs on each side, to work out how many pegs there are around the outside, you would..."

Probing questions

What conclusions can you draw from your evidence? Is it possible to generalise this further?

Is your conclusion based on mathematical explanation or experimental evidence? How do you know?

What if pupils find this a barrier?

Line Crossings

- Draw three straight lines (line segments) so that some cross over each other.
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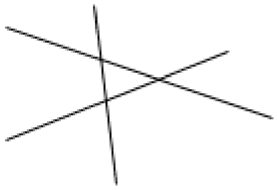

Use the problem **Line Crossings**:

The maximum number of crossings is $(n/2)(n - 1)$, or explained fully without using algebra.

- When you add another line how many extra lines are there? Can you use algebra to explain this?
- Can you explain how you work out how many crossings there are altogether? (If pupils explain 1+2+3+4+5+6, etc., ask if they can explain how they can add this quickly; there are three pairs of seven [1+6, 2+5, 3+4]).