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Progression maps: Using and applying mathematics – Problem solving

Step 1

- Objective: Try different approaches to solve a problem.

Step 2

- Objective: Try different approaches and find ways of overcoming difficulties that arise when solving problems.

Step 3

- Objective: Use a range of strategies when solving problems.

Step 4

- Objective: Develop strategies for solving problems and use these strategies both in working within mathematics and in applying mathematics to practical contexts.

Step 5

- Objective: Begin to structure an approach when exploring a simple task or solving a problem. Generate and check the necessary information.

Step 6

- Objective: Identify the necessary information to carry through tasks and solve mathematical problems. Check results and consider whether they are sensible.

Step 7

- Objective: Solve more complex problems by breaking them into smaller steps or tasks, choosing and using efficient techniques for calculation, algebraic manipulation and graphical representation, and resources, including ICT.

Step 8

- Objective: Solve substantial problems by breaking them into simpler tasks, using a range of efficient techniques, methods and resources, including ICT.

Step 9

- Objective: Starting from given problems or contexts, progressively refine or extend the mathematics used to generate fuller solutions.

Step 10

- Objective: Solve increasingly demanding problems and evaluate solutions; explore connections in mathematics across a range of contexts: number, algebra, shape, space and measures, and handling data.

Step 1 Objective

Try different approaches to solve a problem.

Examples of what pupils should know and be able to do

Final Score investigation

The final score in a football game was 2-1.

- List possible half-time scores.
- How many are there?
- Investigate other final score.

Examples drawn from **Final Score**:

Pupils list some possible half-time scores if the final score is 1-1.

Probing questions

How did you decide what to do?

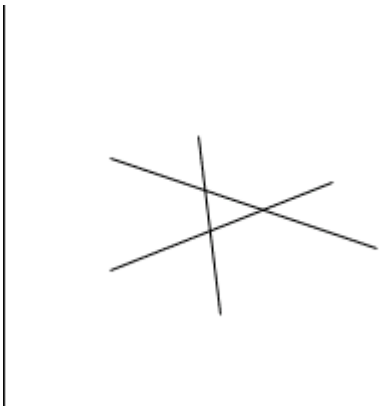
How did you go about exploring/solving this problem?

Could you solve this problem in a different way? How?

What if pupils find this a barrier?

Line Crossings

- Draw three straight lines (line segments) so that some cross over each other.
- How many crossings are there?
- Try different arrangements of the lines. What is the maximum number of possible crossings?
- Try using more lines.
- Is there a rule for the maximum for any number of lines? If so, write it down.



Use the problem **Line Crossings**:

- What happens if you used four (or five) lines instead of the three?
- How did you count the line crossings?

Step 2 Objective

Try different approaches and find ways of overcoming difficulties that arise when solving problems.

Examples of what pupils should know and be able to do

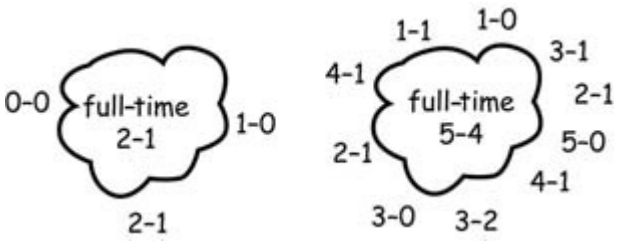
Final Score investigation

The final score in a football game was 2-1

- List possible half-time scores.
- How many are there?
- Investigate other final score.

Examples drawn from **Final Score**:

Pupils produce some half-time scores.



Probing questions

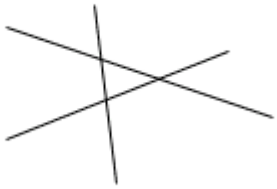
What was the problem that you were trying to solve? What did you try first?

What difficulties did you have? Did you make any changes or do anything differently as a result of these difficulties?

What if pupils find this a barrier?

Line Crossings

- Draw three straight lines (line segments) so that some cross over each other.
- How many crossings are there?
- Try different arrangements of the lines. What is the maximum number of possible crossings?
- Try using more lines.
- Is there a rule for the maximum for any number of lines? If so, write it down.



Use the problem **Line Crossings**:

- If you draw another diagram with four lines do you get the same number of crossings?
- Can you explain?
- What different types of diagram can you draw with four lines?
- How did you count the line crossings?
- Tell me why they are different.

Step 3 Objective

Use a range of strategies when solving problems.

Examples of what pupils should know and be able to do

Final Score investigation

The final score in a football game was 2-1.

- List possible half-time scores.
- How many are there?
- Investigate other final score.

Examples drawn from **Final Score**:

As in Step 2 but with a range of approaches evident.

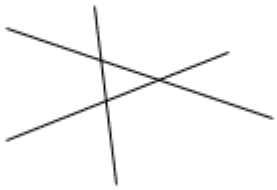
Probing questions

Which different methods did you use?

What if pupils find this a barrier?

Line Crossings

- Draw three straight lines (line segments) so that some cross over each other.
- How many crossings are there?
- Try different arrangements of the lines. What is the maximum number of possible crossings?
- Try using more lines.
- Is there a rule for the maximum for any number of lines? If so, write it down.



Use the problem **Line Crossings**:

- In this diagram you have three (or four or …) lines – how many crossings are there?
- If you add another line to the diagram how many crossings will there be now? Will it be more, the same or fewer?
- Explain why you think so.
- Draw another line. What do you notice?

Step 4 Objective

Develop strategies for solving problems and use these strategies both in working within mathematics and in applying mathematics to practical contexts.

Examples of what pupils should know and be able to do

Final Score investigation

The final score in a football game was 2-1.

- List possible half-time scores.
- How many are there?
- Investigate other final score.

Examples drawn from **Final Score**:

Produce some (but not necessarily all) possible half-time scores for a final score of 2-1 and at least one other final score.

Final score 2-1

There are six possible half-time scores:
1-0, 0-0, 2-0, 1-1, 2-1, 0-1.

Final score 5-3

There are 24 possible half-time scores:
4-2, 1-0, 2-2, 1-1, 0-0, 3-3,
4-1, 4-3, 5-1, 5-2, 5-3, 2-3,
1-2, 1-3, 0-1, 0-2, 0-3.

Final score 3-3

There are 16 possible half-time scores:
0-0, 0-1, 0-2, 0-3, 1-1, 1-2, 1-3,
2-1, 2-2, 2-3, 3-1, 3-2, 3-3.

Probing questions

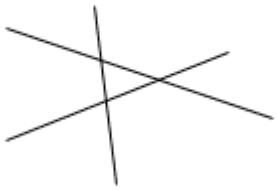
Why did you decide to approach the problem in this way? Tell me how you broke down the problem or calculation into simpler steps. What information did you need to carry out the tasks?

What if pupils find this a barrier?

Line Crossings

- Draw three straight lines (line segments) so that some cross over each other.
- How many crossings are there?

- Try different arrangements of the lines. What is the maximum number of possible crossings?
- Try using more lines.
- Is there a rule for the maximum for any number of lines? If so, write it down.



Use the problem **Line Crossings**:

Look at the different diagrams you have drawn with six (or five or …) lines.

- Which one has the fewest (or most) crossings?
- Explain how you may get more (or fewer) crossings.
- What type of diagram will give you the fewest (or most) crossings for any number of lines?
- What different types of diagram can you draw to get different numbers of crossings?

Step 5 Objective

Begin to structure an approach when exploring a simple task or solving a problem. Generate and check the necessary information.

Examples of what pupils should know and be able to do

Final Score investigation

The final score in a football game was 2-1.

- List possible half-time scores.
- How many are there?
- Investigate other final score.

Examples drawn from **Final Score**:

Find virtually all possible half-time scores for several final scores.

Probing questions

What did you do to find the information needed to solve the problem?

What were your initial thoughts on how to go about exploring this task/solving this problem?

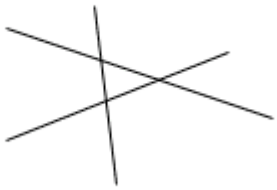
What information did you need to gather?

How did you check your results/findings?

What if pupils find this a barrier?

Line Crossings

- Draw three straight lines (line segments) so that some cross over each other.
- How many crossings are there?
- Try different arrangements of the lines. What is the maximum number of possible crossings?
- Try using more lines.
- Is there a rule for the maximum for any number of lines? If so, write it down.



Use the problem **Line Crossings**:

- Are you sure that you have all of the different arrangements for four (or five) lines?
- How can you be sure?
- What different types of arrangement are there for five or six lines?

Step 6 Objective

Identify the necessary information to carry through tasks and solve mathematical problems. Check results and consider whether they are sensible.

Examples of what pupils should know and be able to do

Final Score investigation

The final score in a football game was 2-1.

- List possible half-time scores.
- How many are there?
- Investigate other final score.

Examples drawn from **Final Score**:

Use a system that consistently produces all possible half-time scores.

2-0	2-2	2-3	2-4	2-5
0-0	2-1	2-2	2-3	2-4
1-0	1-1	2-1	2-2	2-3
Total 3	1-0	2-0	2-1	2-2
	0-1	1-3	1-4	2-1
	0-0	1-2	1-3	2-0
	1-2	1-1	1-2	1-5
	0-2	1-0	1-1	1-4
	2-0	0-1	0-4	1-3
	Total 9	0-0	0-3	1-2
		0-3	0-2	1-1
		0-3	0-1	1-0
		Total 12	0-0	0-5
			2-0	0-4
			1-0	0-3
			Total 15	0-2
				0-1
				0-0
				Total 18

Probing questions

What was the most important information that you needed to use?

How did you check your results/findings?

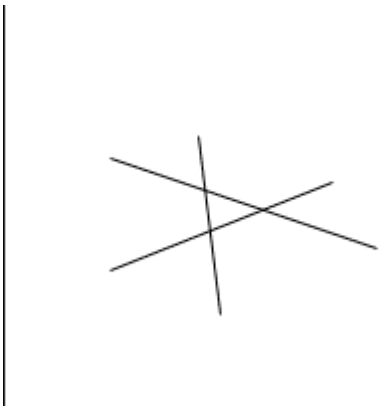
Do your results/findings seem reasonable?

How do you know?

What if pupils find this a barrier?

Line Crossings

- Draw three straight lines (line segments) so that some cross over each other.
- How many crossings are there?
- Try different arrangements of the lines. What is the maximum number of possible crossings?
- Try using more lines.
- Is there a rule for the maximum for any number of lines? If so, write it down.

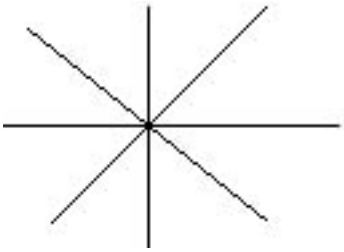


Use the problem **Line Crossings**:

Look at all of the diagrams with five (or six) lines:

- How could you organise your results so that they go from smallest to largest number of crossings?
- What patterns can you see?
- Do you think that there are any mistakes? (Or explain why you think your results are correct.)
- What happens when more than two lines cross at the same point?

How many crossings are there here, one, three or six? Explain.



Step 7 Objective

Solve more complex problems by breaking them into smaller steps or tasks, choosing and using efficient techniques for calculation, algebraic manipulation and graphical representation, and resources, including ICT.

Examples of what pupils should know and be able to do

Final Score investigation

The final score in a football game was 2-1.

- List possible half-time scores.
- How many are there?
- Investigate other final score.

Examples drawn from **Final Score**:

As in Step 6 half-time scores, working independently and structuring their work.

Probing questions

What did you see as the main steps in solving this problem/working on this task?

How did you go about organising your approach? Did you need to make any changes to your planned approach?

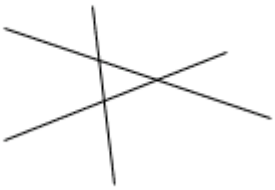
Select from questions below as appropriate to context:

- How did your use of algebra help?
- How did your use of graphs/tables help?
- Why did you use these techniques for calculation? Are there other ways? Are these more efficient?

What if pupils find this a barrier?

Line Crossings

- Draw three straight lines (line segments) so that some cross over each other.
- How many crossings are there?
- Try different arrangements of the lines. What is the maximum number of possible crossings?
- Try using more lines.
- Is there a rule for the maximum for any number of lines? If so, write it down.



Use the problem **Line Crossings**:

Look at the number of crossings for three, four, five, six lines:

- Which results are the best ones to compare? (Greatest number of crossings.)
- What is the best way to show this information, to compare the number of crossings?

Step 8 Objective

Solve substantial problems by breaking them into simpler tasks, using a range of efficient techniques, methods and resources, including ICT.

Examples of what pupils should know and be able to do

Final Score investigation

The final score in a football game was 2-1.

- List possible half-time scores.
- How many are there?
- Investigate other final score.

Examples drawn from **Final Score**

Look at a set of related scores - e.g. 0-1, 1-1, 2-1, 3-1, 4-1, 5-1 and consider pattern in results.

Full-time score for draws	0-0	1-1	2-2	3-3
H-T	0-0	1-0	2-0	3-0
	1	0-1	2-1	3-1
		0-0	2-2	3-2
		1-1	1-0	3-3
		4	1-1	2-0
			1-2	2-1
			0-0	2-2
			0-1	2-3
			0-2	1-0
			9	1-1
				1-2
				1-3
				0-0
				0-1
				0-2
				0-3
				16

The number of half-time scores for draws goes up in a pattern 3, 5, 7. It gets two bigger each time.

Result	No. of Half-times
0-0	1
	-> 3
1-1	4
	-> 5
2-2	9
	-> 7
3-3	16

Probing questions

What do you think makes this a substantial problem to solve/task to explore? When did you realise the potential of this problem/task?

What did you see as the main steps in solving this problem/working on this task?

How did you go about organising your approach? Did you need to make any changes to your planned approach?

What resources, including ICT, helped you to explore this problem/task? How?

What if pupils find this a barrier?

Line Crossings

- Draw three straight lines (line segments) so that some cross over each other.
- How many crossings are there?
- Try different arrangements of the lines. What is the maximum number of possible crossings?
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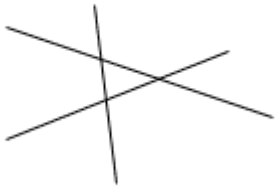

Use the problem **Line Crossings**:

How did you draw your line patterns to make sure that you had the maximum number of crossings?

Look at the table of results:

- Is there a good pattern or not?
- How would the result table help to show if you had made any mistakes?
- How would you check if your predictions are correct?

Step 9 Objective

Starting from given problems or contexts, progressively refine or extend the mathematics used to generate fuller solutions.

Examples of what pupils should know and be able to do

Final Score investigation

The final score in a football game was 2-1.

- List possible half-time scores.
- How many are there?
- Investigate other final score.

Examples drawn from **Final Score**:

Pupils extend the Step 8 example by considering the results for any given full-time score.

Probing questions

How did you extend your task?

How did you use the answer to the initial problem to help solve the fuller problem?

Explain how your final solution can be used to solve your first problem.

What if pupils find this a barrier?

Line Crossings

- Draw three straight lines (line segments) so that some cross over each other.
- How many crossings are there?
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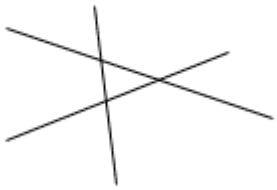


Use the problem **Line Crossings**:

- How would you work out the number of crossings if you have an extra line in your diagram (but you are not allowed to draw the lines.)

Step 10 Objective

Solve increasingly demanding problems and evaluate solutions; explore connections in mathematics across a range of contexts: number, algebra, shape, space and measures, and handling data.

Examples of what pupils should know and be able to do

Final Score investigation

The final score in a football game was 2-1.

- List possible half-time scores.
- How many are there?
- Investigate other final score.

Examples drawn from **Final Score**:

Pupils can predict the sequence of results for the total number of half-time scores.

They make explicit a rule for calculating the total number of possible half-time scores for a given final score.

They can write the rule algebraically.

The formula for finding the number of half time scores is:

Add 1 to each score and multiply the two resulting numbers together.

Example

If the score was 3-2 (3 + 1) x (2 + 1) = 4 x 3 = 12

If the score was m-n (m + 1)(n + 1)

Probing questions

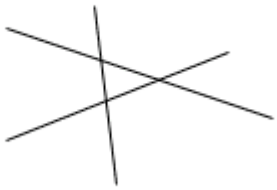
What did you find out in solving this problem/exploring this task?

Did you notice any links with other work you have done?

What if pupils find this a barrier?

Line Crossings

- Draw three straight lines (line segments) so that some cross over each other.
- How many crossings are there?
- Try different arrangements of the lines. What is the maximum number of possible crossings?
- Try using more lines.
- Is there a rule for the maximum for any number of lines? If so, write it down.



Use the problem **Line Crossings**:

Ask the pupil to look back at the earlier solutions and explain how the patterns are developing. Then ask them to extend the problem as was started in Step 9 Line Crossings.
