

## Progression maps: Using and applying mathematics – Communicating

### Step 1

- Objective: Discuss work, using mathematical language. Represent work, using symbols and simple diagrams.

### Step 2

- Objective: Begin to organise work. Use and interpret mathematical symbols and diagrams.

### Step 3

- Objective: Begin to refine ways of recording and use appropriate mathematical symbols correctly.

### Step 4

- Objective: Present information and results in a clear and organised way. Present solutions/findings in the context of the problem/task.

### Step 5

- Objective: Present and interpret solutions/findings in the context of the problem/task. Begin to develop correct and consistent use of notation, symbols and diagrams.

### Step 6

- Objective: Show understanding of situations by describing them mathematically, making correct use of symbols, words, diagrams, tables and graphs.

### Step 7

- Objective: Choose and use correctly symbols, diagrams and graphs. Present and interpret solutions/findings in the context of the original problem/task.

### Step 8

- Objective: Interpret, discuss and synthesise information presented in a variety of mathematical forms. Begin to explain reasons for selection and use of diagrams.

### Step 9

- Objective: Represent problems and synthesise information in algebraic, geometric or graphical form; move from one form of presentation to another to gain a different perspective on the problem/task.

### Step 10

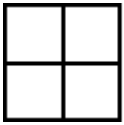
- Objective: Examine critically, improve and justify the choice of mathematical presentation, explaining features selected.

## Step 1 Objective

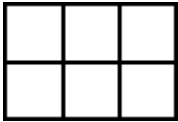
Discuss work using mathematical language. Represent work using symbols and simple diagrams.

## Examples of what pupils should know and be able to do

### Shading Squares



There are six different ways to shade two squares in this shape. Can you find them all?



What about this shape? How many ways are there?

Try using different rectangles made up of more squares.

Try shading three squares.

Example drawn from **Shading Squares**:

Try shading these squares.

Draw at least three different ways of shading two squares out of four.

And then draw all six ways of shading two squares out of four.



### Probing questions

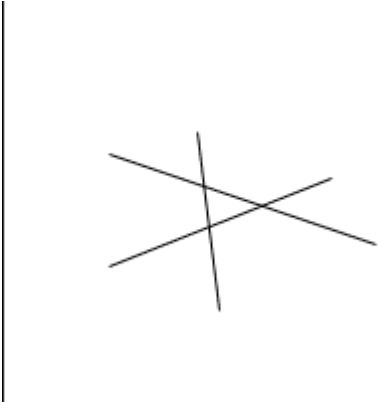
What have you noticed?

Tell me what the diagram(s)/results are showing.

### What if pupils find this a barrier?

#### Line Crossings

- Draw three straight lines (line segments) so that some cross over each other.
- How many crossings are there?
- Try different arrangements of the lines. What is the maximum number of possible crossings?
- Try using more lines.
- Is there a rule for the maximum for any number of lines? If so, write it down.



Use the problem **Line Crossings**:

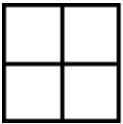
- Draw other diagrams using only four (or five) lines; how many different numbers of intersections (crossings) can you get?
- How many more diagrams with a different number of intersections can you draw using four (or five) lines?
- How might you record the maximum number of crossings for three, four, five lines?
- Are there other ways?

## Step 2 Objective

Begin to organise work. Use and interpret mathematical symbols and diagrams.

### Examples of what pupils should know and be able to do

#### Shading Squares



There are six different ways to shade two squares in this shape. Can you find them all?



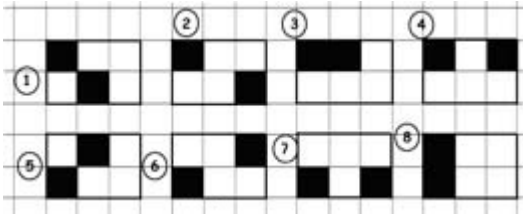
What about this shape? How many ways are there?

Try using different rectangles made up of more squares.

Try shading three squares.

Examples drawn from **Shading Squares**:

Show evidence of using a systematic approach in finding several pairs when shading two squares out of 6.



## Probing questions

Why did you...?

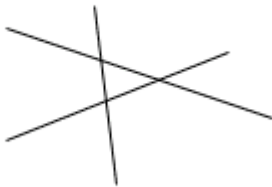
How did your diagrams/table/graph help?

What other diagrams did you think of using?

## What if pupils find this a barrier?

### Line Crossings

- Draw three straight lines (line segments) so that some cross over each other.
- How many crossings are there?
- Try different arrangements of the lines. What is the maximum number of possible crossings?
- Try using more lines.
- Is there a rule for the maximum for any number of lines? If so, write it down.



Use the problem **Line Crossings**:

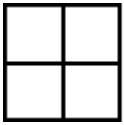
- For each diagram explain how many lines and how many crossings there are. Write this down next to each diagram.
- What is the best way of writing this information down?

## Step 3 Objective

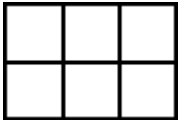
Begin to refine ways of recording and use appropriate mathematical symbols correctly.

## Examples of what pupils should know and be able to do

### Shading Squares



There are six different ways to shade two squares in this shape. Can you find them all?



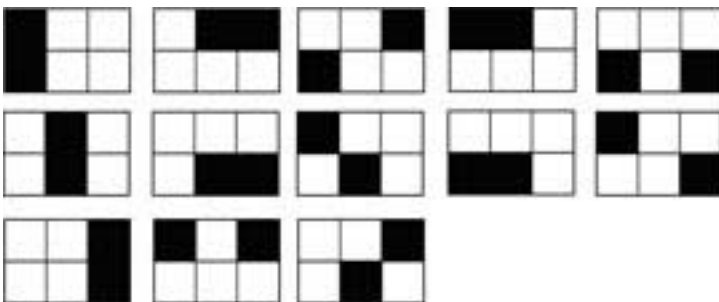
What about this shape? How many ways are there?

Try using different rectangles made up of more squares.

Try shading three squares.

Examples drawn from **Shading Squares**:

Find 10 or more of the arrangements for two squares out of six without any repeats.



### Probing questions

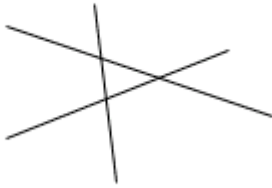
Which methods of organising information have you found most helpful?

How is this different from the way you have recorded information before?

### What if pupils find this a barrier?

#### Line Crossings

- Draw three straight lines (line segments) so that some cross over each other.
- How many crossings are there?
- Try different arrangements of the lines. What is the maximum number of possible crossings?
- Try using more lines.
- Is there a rule for the maximum for any number of lines? If so, write it down.



Use the problem **Line Crossings**:

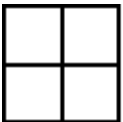
- Draw other diagrams using only four (or five) lines; how many different numbers of intersections (crossings) can you get?
- How many more diagrams with a different number of intersections can you draw using four (or five) lines?
- How might you record the maximum number of crossings for three, four, five lines?
- Are there other ways?

## Step 4 Objective

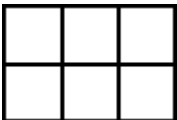
Present information and results in a clear and organised way. Present solutions/findings in the context of the problem/task.

## Examples of what pupils should know and be able to do

### Shading Squares



There are six different ways to shade two squares in this shape. Can you find them all?



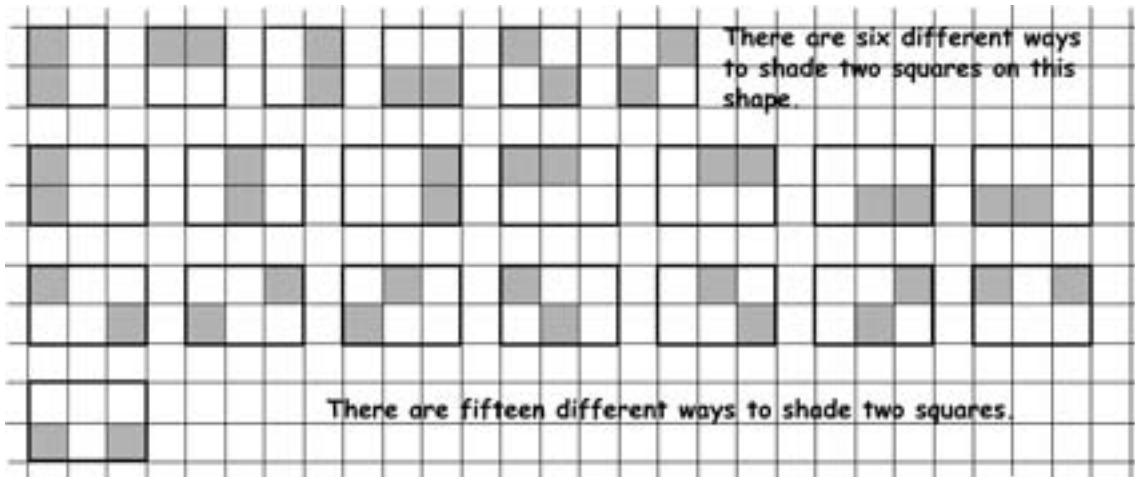
What about this shape? How many ways are there?

Try using different rectangles made up of more squares.

Try shading three squares.

Examples drawn from **Shading Squares**:

State there are 15 ways of shading 2 squares out of 6 without repeats.



## Probing questions

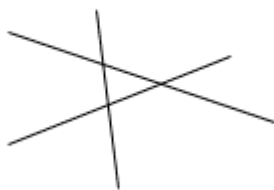
Which ways of organising information have you found most helpful? How does it help?

What have you found out?

## What if pupils find this a barrier?

### Line Crossings

- Draw three straight lines (line segments) so that some cross over each other.
- How many crossings are there?
- Try different arrangements of the lines. What is the maximum number of possible crossings?
- Try using more lines.
- Is there a rule for the maximum for any number of lines? If so, write it down.



Use the problem **Line Crossings**:

You have drawn different patterns, using three (or four) lines. It gets more complicated when you use even more lines.

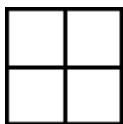
- Draw different diagrams for five (or six) lines. Start with those where there are only a few crossings (intersections).
- How can you show that your work is organised?

## Step 5 Objective

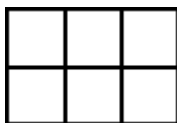
Present and interpret solutions/findings in the context of the problem/task. Begin to develop correct and consistent use of notation, symbols and diagrams.

## Examples of what pupils should know and be able to do

### Shading Squares



There are six different ways to shade two squares in this shape. Can you find them all?



What about this shape? How many ways are there?

Try using different rectangles made up of more squares.

Try shading three squares.

Examples drawn from **Shading Squares**:

In the diagram for Step 4, the pupils can explain their system to ensure that they do not have repeats.

### Probing questions

What have you found out?

Why do you think this is?

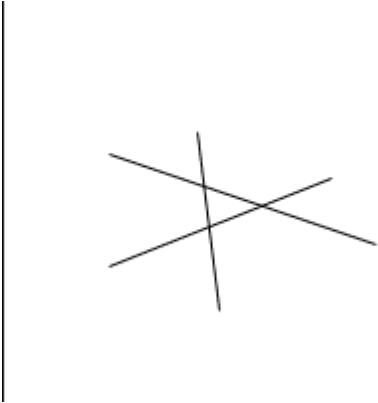
How is your use of mathematical notation and symbols improving? Show me some examples.

How is your use of diagrams improving? Show me some examples.

### What if pupils find this a barrier?

#### Line Crossings

- Draw three straight lines (line segments) so that some cross over each other.
- How many crossings are there?
- Try different arrangements of the lines. What is the maximum number of possible crossings?
- Try using more lines.
- Is there a rule for the maximum for any number of lines? If so, write it down.



Use the problem **Line Crossings**:

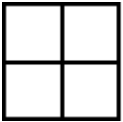
- What can you say (see) from the diagrams with four (or five) lines?
- What do you call the lines that don't cross each other?
- How would you explain what happens when there are a lot of crossings?

## Step 6 Objective

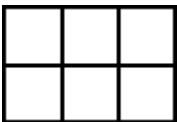
Show understanding of situations by describing them mathematically making correct use of symbols, words, diagrams, tables and graphs.

## Examples of what pupils should know and be able to do

### Shading Squares



There are six different ways to shade two squares in this shape. Can you find them all?



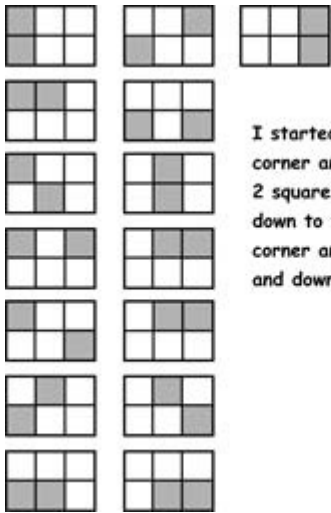
What about this shape? How many ways are there?

Try using different rectangles made up of more squares.

Try shading three squares.

Examples drawn from **Shading Squares**:

Develops and uses a procedure which guarantees to produce all the ways of shading two squares in a given number of squares.



I started in the top left hand corner and try to make as many 2 squares as could then I went down to the bottom left hand corner and carried on going up and down so I found every one.

## Probing questions

What is happening here?

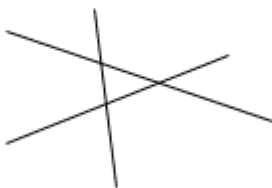
What is this diagram, table, graph showing you?

How have you expressed this mathematically?

## What if pupils find this a barrier?

### Line Crossings

- Draw three straight lines (line segments) so that some cross over each other.
- How many crossings are there?
- Try different arrangements of the lines. What is the maximum number of possible crossings?
- Try using more lines.
- Is there a rule for the maximum for any number of lines? If so, write it down.



Use the problem **Line Crossings**:

You have drawn a lot of diagrams.

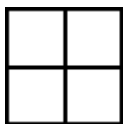
- Can you show the results for different numbers of lines together?
- What information would you put into a table of results? Can you draw this table?

## Step 7 Objective

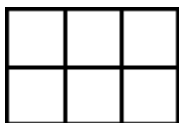
Choose and use correctly symbols, diagrams and graphs. Present and interpret solutions/findings in the context of the original problem/task.

## Examples of what pupils should know and be able to do

### Shading Squares



There are six different ways to shade two squares in this shape. Can you find them all?



What about this shape? How many ways are there?

Try using different rectangles made up of more squares.

Try shading three squares.

Examples drawn from **Shading Squares**:

As in Step 6, the pupils can work systematically and then refine the written explanation so that it is clear what the diagrams are showing.

### Probing questions

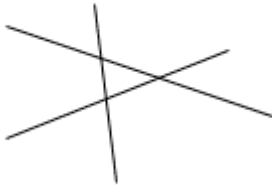
Why did you decide to use this diagram/graph? Did you discount other forms of diagram/graph? Why is this form better?

How has your use of notation and symbols helped you to record your findings?

### What if pupils find this a barrier?

#### Line Crossings

- Draw three straight lines (line segments) so that some cross over each other.
- How many crossings are there?
- Try different arrangements of the lines. What is the maximum number of possible crossings?
- Try using more lines.
- Is there a rule for the maximum for any number of lines? If so, write it down.



Use the problem **Line Crossings**:

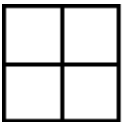
- What does the table of results show?
- Can you write this down?
- How does it help us to see whether there are incorrect results?

## Step 8 Objective

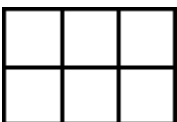
Interpret, discuss and synthesise information presented in a variety of mathematical forms. Begin to explain reasons for selection and use of diagrams.

### Examples of what pupils should know and be able to do

#### Shading Squares



There are six different ways to shade two squares in this shape. Can you find them all?



What about this shape? How many ways are there?

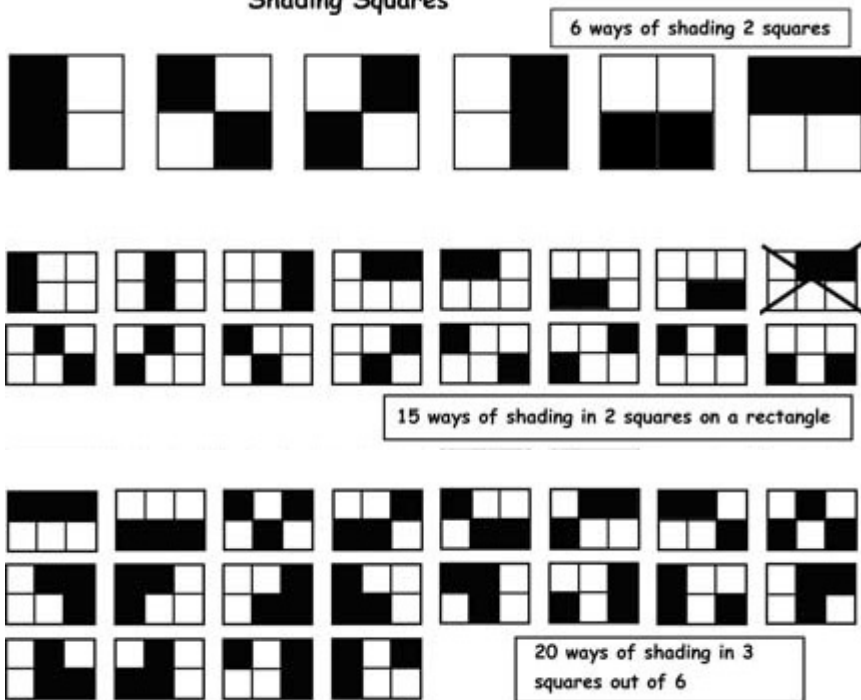
Try using different rectangles made up of more squares.

Try shading three squares.

Examples drawn from **Shading Squares**:

Produce all the ways of shading two squares out of four, six and eight squares – may develop to three squares out of six, etc.

## Shading Squares



## Probing questions

How would you explain to someone else what your results mean?

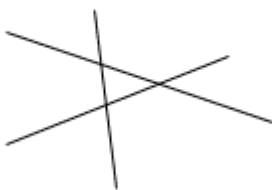
Is all the information you have gathered here useful?

Why did you decide to use these graphs/diagrams/tables in this way? How have they helped to further your work? How has your use of notation, diagrams, tables and/or graphs helped you to communicate your results?

## What if pupils find this a barrier?

### Line Crossings

- Draw three straight lines (line segments) so that some cross over each other.
- How many crossings are there?
- Try different arrangements of the lines. What is the maximum number of possible crossings?
- Try using more lines.
- Is there a rule for the maximum for any number of lines? If so, write it down.



Use the problem **Line Crossings**:

Look at your table of results:

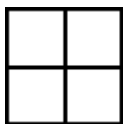
- Explain to me what it shows and what it means. How would you write this down?
- How would these results help you to explain what happens when you add another line?

## Step 9 Objective

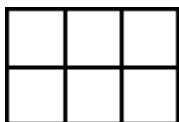
Represent problems and synthesise information in algebraic, geometric or graphical form; move from one form of presentation to another to gain a different perspective on the problem/task.

### Examples of what pupils should know and be able to do

#### Shading Squares



There are six different ways to shade two squares in this shape. Can you find them all?



What about this shape? How many ways are there?

Try using different rectangles made up of more squares.

Try shading three squares.

Examples drawn from **Shading Squares**:

Pupils explain why they have chosen a particular presentation. The presentation may be symbolic or diagrammatic, as with the Shading Squares activity.

#### Probing questions

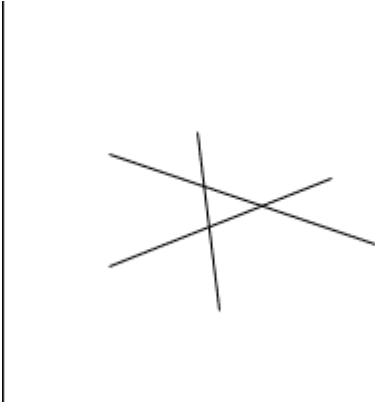
How does this representation link to this other one? What does each tell you?

What does this form tell you that the other forms cannot?

#### What if pupils find this a barrier?

#### Line Crossings

- Draw three straight lines (line segments) so that some cross over each other.
- How many crossings are there?
- Try different arrangements of the lines. What is the maximum number of possible crossings?
- Try using more lines.
- Is there a rule for the maximum for any number of lines? If so, write it down.



Use the problem **Line Crossings**:

Look at your table of results:

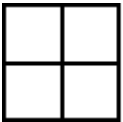
- Can you explain a rule for finding the number of intersections for any number of lines?
- How would you write this down?

## Step 10 Objective

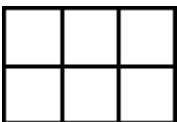
Examine critically, improve and justify the choice of mathematical presentation, explaining features selected.

### Examples of what pupils should know and be able to do

#### Shading Squares



There are six different ways to shade two squares in this shape. Can you find them all?



What about this shape? How many ways are there?

Try using different rectangles made up of more squares.

Try shading three squares.

Examples drawn from **Shading Squares**:

Develops and uses a procedure guaranteeing all of the ways of shading two squares in a given number of squares, making observations based on correct results. They can systematically produce results for shading squares without drawing the diagrams.

4 Squares

1 2 3	1 4 5
2 4 6	3 5 6

(6)

14

1 2 3 4 5 6 7 8
9 10 11 12 13 14 15 16

(91)

3 x 2

1 4 9	2 4 8	3 7 15
14 15	10 15	14 17
16		
15 10	2 5 7 9	3 8 11
11 16 17	12	12 15

(15) 8

4 x 2

1 2 3 4	1 8 9	2 8 14	3 9 14
5 6 7	10 11	15 16	19 20
	12 13	17 18	21 22
7 15 15	6 12 16	5 11 17	4 10 12
19 25	20 28	21 24	22 15
24 25	27	26 28	27 28

(28)

## Probing questions

What forms of presentation have you used to help analyse information and communicate findings?

What sort of improvements to your initial attempts have you made?

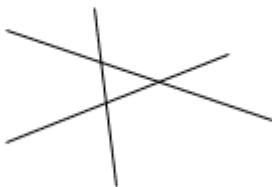
How did they help you to solve the problem/develop your enquiry further?

What have you found out that will be useful in future work?

## What if pupils find this a barrier?

### Line Crossings

- Draw three straight lines (line segments) so that some cross over each other.
- How many crossings are there?
- Try different arrangements of the lines. What is the maximum number of possible crossings?
- Try using more lines.
- Is there a rule for the maximum for any number of lines? If so, write it down.



Use the problem **Line Crossings**:

(Pupils have rule  $(n/2)(n - 1)$  or the rule in words.)

- That is a good rule; explain to me what it means.
- Why is this rule more useful than drawing diagrams and counting intersections?

(Pupils have explained that for every extra line, the number of extra crossings is one less than the new total number of lines.)

- Can you explain why this (algebraic) rule is more helpful in finding the number of crossings than just adding the number of extra crossings each time? Can you write that down?

