

## Key processes in number

The foundations of the number system and processes of calculation are laid in Key Stages 1 and 2, and Key Stages 3 and 4 need to build on these foundations. Effective calculation is based on secure understanding of the number line, place value, relationships between operations and the laws of arithmetic (including the distributive law) and knowledge of number facts, together with strategies to quickly re-calculate any forgotten facts.

After calculation, the application of proportional reasoning is the most important aspect of elementary number. Proportionality underlies key aspects of number (ratios and rates, percentages and proportions), algebra (linearity, gradient), geometry and measures (enlargement, similarity, trigonometry) and statistics (proportions and probability). Being able to think multiplicatively as well as additively is a major shift that pupils need to make if they are not to remain dependent on informal approaches.

## Representing

Representing a situation places it into a mathematical form that enables it to be worked on. This means more than just extracting the relevant numbers: it involves exploration of methods and tools and making choices between equivalent forms. Aspects of representing within number include:

- developing understanding of the number line and of tabular arrays, such as a place-value chart
- recognising the broader use of the equals sign to express the equivalence of two numbers or expressions
- choosing appropriate numerical representations, using knowledge of equivalent forms (e.g. of fractions, decimals and percentages)
- identifying the type of problem and the operations needed to reach a solution
- choosing between mental, written and calculator methods
- identifying the numerical data relevant to solving a problem and representing it to an appropriate degree of accuracy
- organising data in a suitable format, for example, a labelled  $2 \times 2$  array representing a proportion
- breaking down more complex problems into a sequence of steps.

Simple problems are often solved informally, particularly where the numbers involved are easy to deal with mentally. Pupils also need to encounter more difficult, realistic and challenging problems, requiring methods that are more generally applicable, if they are to develop functional skills in number.

## Analysing – use mathematical reasoning

Reasoning is involved when representing a numerical problem or situation, when calculating and when interpreting results. It includes:

- making and using connections between the number operations, for example, that indices represent repeated self-multiplication or that dividing by a number is the same as multiplying by its reciprocal
- making connections between fractions and division

- making connections between additive methods of proportional change and those using a single multiplier
- working logically to choose, use and review techniques to solve a multi-step problem
- using different techniques to analyse a situation, evaluate an approach or check a calculation, for example, looking at simpler cases or working backwards
- identifying, classifying and generalising numerical patterns.

Ultimately, reasoning in number will lead into algebra. As the two strands develop in tandem, pupils are in a position to realise that explaining and convincing within number becomes a constraint and that proving and generalising requires a symbolic form and the discipline and structure of algebra.

## **Analysing – use appropriate mathematical procedures**

Using appropriate procedures involves manipulating numbers, using and applying algorithms and techniques, and monitoring the accuracy of methods and solutions.

Appropriate procedures in number include:

- building on the approach to calculation developed in Key Stages 1 and 2, which emphasises mental methods and written methods that are gradually refined and made efficient
- visualising images, such as a number line or other representation, to support mental methods
- extending procedures to calculations with fractions, decimals and percentages
- developing efficient use of calculators and a spreadsheet (e.g. for optimisation problems)
- developing multiplicative methods for solving problems involving direct proportion, extending to inverse proportion
- developing routines for estimating, approximating and checking calculations.

When entering Key Stage 3, pupils need opportunities to discuss their mental methods of calculation, to explore and evaluate different approaches to particular calculations, and to practise and extend their mental fluency. They also need opportunities to discuss their written methods of calculation, including expanded methods. When refining methods (e.g. developing column procedures), they should do so with consideration for understanding and accuracy as well as efficiency. They should learn when it is appropriate to use a calculator and how to use a basic calculator, and later a scientific calculator, accurately and efficiently. Mental methods of approximating, estimating and checking are an important adjunct to all calculation.

## **Interpreting and evaluating**

Aspects of interpreting and evaluating in number include:

- considering the appropriateness and accuracy of numerical results
- relating numerical results to the problem or context under consideration and drawing conclusions
- considering the effectiveness of different strategies, for example, when completing a mental calculation
- following the logic of someone else's calculation, particularly the effects of rounding

- appreciating when to move to algebraic form rather than elaborating numerical examples.

## Communicating and reflecting

Aspects of communicating and reflecting of particular relevance to number include:

- using precise language and symbolism to set out solutions in an organised and understandable way
- discussing and reflecting on different approaches to solving a numerical problem
- linking mental methods with jottings and representations, for example, on a number line
- comparing the efficiency of calculation procedures
- considering the effects of rounding.

## Resources for number

A range of resources to support the development of key processes within number are included in the 'Ideas for rich tasks' folder within the *Secondary mathematics planning toolkit*:

- *Strengthening mathematics teaching and learning in Year 7*
- *Interacting with mathematics in Key Stage 3 – Proportional reasoning:*
  - *Year 7 fractions and ratio*
  - *Year 8 multiplicative relationships*
  - *Year 9 proportional reasoning*
  - *Enhancing PR in Y8 and Y9*
  - *PR transition lessons*
- *Teaching mental mathematics from level 5:*
  - *Number*
  - *Measures and mensuration in number*
- Standards Unit *Improving learning in mathematics:*
  - *Mostly number* (Sessions N1–N13).