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ASSESSMENT and
QUALIFICATIONS
ALLIANCE

General Certificate of Secondary Education
Functional Skills Certificate

Mathematics 9307
Functional Mathematics 9305

Pilot Specifications
2008

TEACHER'S GUIDE AND TEACHING RESOURCE

Further copies of this booklet are available from:

The GCSE Mathematics Department, AQA, Devas Street, Manchester, M15 6EX
Telephone: 0161 957 3852 Fax: 0161 957 3873

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Dr Michael Cresswell, Director General.

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Background Information

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Introduction

1.1 Purpose

This resource has been written by Leeds University's Assessment Evaluation Unit to support teachers in developing approaches to the type of task that will appear in the AQA pilot assessment of Functional Mathematics.

This Teacher's Guide is intended to be equally useful to teachers preparing candidates for the pilot GCSE Mathematics Specification 9307 and those who are following a stand-alone Functional Mathematics course for the pilot Functional Skills Specification 9305. The materials are aimed at Level 2 of the functional skills standards but should be suitable for preparing students working towards Level 1.

As the subject content and style of Units 2 and 3 of the pilot GCSE specification are the same as in the present AQA GCSEs in mathematics, the Teacher's Guide produced to support AQA specifications 4301 and 4302 will remain useful to teachers of this specification.

The guide should be read in conjunction with the specification documents and the specimen materials that accompany them. The specifications and specimen assessment materials are also available from the GCSE Mathematics Department, AQA, Devas Street, Manchester, M15 6EX, Telephone: 0161 957 3852, Fax: 0161 957 3873

1.2 Introduction

This resource is designed to support the teaching and preparation for the AQA Pilot assessment of Functional Mathematics.

The resource consists of:

1. these introductory pages, about:
    (a) functional mathematics and its assessment in the new examinations
    (b) a description of a possible strategy for teaching functional mathematics
2. 30 examples of functional mathematics items, including both 'data sheets' and questions
3. descriptions of the features of 10 of the items;
4. the answers to the questions

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Specifications at a Glance *Mathematics and Functional Mathematics*

2.1 GCSE Mathematics

- This is one of two pilot GCSE Mathematics specifications offered by AQA. This specification is a modular specification leading to a GCSE in Mathematics and a separate Functional Mathematics qualification. There is also a pilot specification for GCSE in Additional Mathematics which is a linear specification.
- A Level 2 assessment of Functional Mathematics is a requirement of this pilot specification. This unit must be entered but the achievement in this unit is awarded separately as Level 2, Level 1 or Unclassified, and does not contribute to the overall grading of GCSE mathematics.
- There are two tiers of assessment for GCSE, Foundation (C – G) and Higher (A* – D).

GCSE Mathematics Pilot (9307)

Functional Mathematics (FM)

Paper 1 (Non-calculator)	**Competency Test**
Written Paper	2/7 of the assessment of Functional Mathematics
40 minutes (Untiered)	
Paper 2 (Calculator)	**Functionality Test**
Written Paper with pre-release material	5/7of the assessment of Functional Mathematics
1 hour 15 minutes (Untiered)	

Unit 2

Number and Statistics

Written Paper	1/3 of the total GCSE Mathematics assessment
2 × 30 minutes (Both tiers)	
Section A – Calculator Section B – Non-calculator	

Unit 3 **Geometry and measures and Algebra**	
2 Written Papers	2/3 of the total GCSE Mathematics assessment
60 minutes each (Both tiers)	
Paper 1 – Non-calculator Paper 2 – Calculator	

2.2 Functional Skills Certificate

- This is the sole pilot stand-alone Functional Mathematics specification offered by AQA

	Functional Mathematics Pilot (9305)	
Level 1	Written Paper with pre-release material	
	Section A (Calculator)	Functionality 3/4 of the assessment
	Section B (Non-calculator)	Competency 1/4 of the assessment
	1 hour 15 minutes	
Level 2	Paper 1 (Non-calculator) Written Paper	**Competency** 2/7 of the assessment
	40 minutes	
	Paper 2 (Calculator) Written Paper with pre-release material	**Functionality** 5/7of the assessment
	1 hour 15 minutes	

3 Assessment Issues

3.1 Functional Mathematics and its assessment

The White Paper "14-19 Education and Skills" (DfES, 2005) defines Functional Mathematics as the "maths that people need to participate effectively in everyday life, including in the workplace" (page 35) and expresses the intention to set up a system in which Functional Mathematics can be assessed – the one which is currently being piloted.

It is proposed that Functional Mathematics will be assessed as a stand-alone qualification, and a number of pilots are under way in relation to this. It is expected that a 'pass' in Functional Mathematics will eventually be a hurdle to the award of GCSE in Mathematics at grade C or above, although not in the pilot phase, and also the award of diplomas at level 2.

Consideration of what exactly Functional Mathematics could and should be has resulted in a set of Standards, which outline the framework for the qualification. See:

http://www.qca.org.uk/downloads/QCA-06-2932_Functional_skills_standards_maths.pdf

At Level 2, which is approximately equivalent to GCSE grade C, these state that students should be able to:

- understand routine and non-routine problems in a wide range of familiar and unfamiliar contexts and situations;
- identify the situation or problem and the mathematical methods needed to tackle it;
- select and apply a range of mathematics to find solutions;
- use appropriate checking procedures and evaluate their effectiveness at each stage;
- interpret and communicate solutions to practical problems in familiar and unfamiliar routine contexts and situations;
- draw conclusions and provide mathematical justifications.

The AQA Pilot Unit in Functional Mathematics takes these to imply that students will need to demonstrate that they can apply mathematics in contexts. However, 'word-problems' can be too stereotyped and familiar to assess effectively students' application skills. More substantial descriptions of situations are needed, which students will have to make sense of and engage with. When questions that require mathematics to be deployed are asked about these more complex contexts, the skills used will be closer to those used by adults when being mathematically functional.

This resource contains 30 examples of data sheets describing situations, and questions to be asked about them, that reflect the form of the items used in the examination. As detailed in the following pages, the resource can be used in a programme of teaching and preparation for the Pilot examination.

3.2 The assessment of Functional Mathematics in the AQA Pilot GCSE

The AQA Pilot GCSE consists of three units. Two of these (Units 2 and 3) are similar to those in the current GCSE and comprise the total GCSE assessment of Functional Mathematics is assessed through two externally-assessed examination papers.

The first and smaller of the Functional Mathematics papers in the AQA Pilot for GCSE Mathematics is a 40-minute, 30-question test of one-mark items that covers the key Mathematics knowledge and skills necessary for being functional. This is referred to as a 'competency test'.

The second, larger, Functional Mathematics paper is a 'functionality test' that includes data sheets describing situations, and a set of questions about the situations that require the use of mathematics. In the pilot examinations, five situations are given as 'data sheets', two of which will be issued as 'pre-release' material, to be studied before the examination itself. This enables more complex and authentic 'situations' to be described than would be possible if they were seen for the first time in the exam.

Neither part of the assessment of Functional Mathematics will draw on mathematics that is additional to that which is already being learned for GCSE. However, each examination has demands that are different from those in the current GCSE, and awareness of these is necessary to ensure that students are properly prepared.

The competency test

In the case of the competency test, the content of the test is set at a relatively modest level (below GCSE level 'C' in the case of the Level 2 test) with 30 straightforward questions rehearsing simple items of knowledge and skill, generally without context. The challenge in this assessment is a performance that demonstrates 'mastery' – ie, students need to get most of them right. For many students, the individual questions will be seen as easy, but that does not necessarily mean that they will find it easy to get most of them right in a timed test. In many cases the skills will not have been practised recently, and may be a little 'rusty'. This is not a significant barrier to success, however, and will probably just require some attention in the form of revision and revitalisation before students take the examination.

The functionality test

A greater challenge to successful Functional Mathematics performance on the basis of existing knowledge is presented by the second Functional Mathematics paper, in which there are descriptions of situations and questions about them.

There are generally about five described contexts in the examination, two of which will have the descriptions circulated in advance as pre-release material. The contexts described could come from any aspect of 'real-life' that has mathematical elements, including work, leisure, domestic activities, commerce, industry and so on. Each item has between three and seven mathematical questions about the described context.

For most items, the questions are 'ramped' in difficulty, starting with the easiest, often a straightforward reading of data from the information given. Later questions require more to be done with the data, through calculation, manipulation or combination of data, sometimes following the introduction of more data. Often the items end with a more challenging question that may be multi-step or combine different areas of mathematics.

There are many students who have a strong understanding of mathematical ideas 'out of context' who do not at first successfully use and apply those ideas when the mathematics is in context. It is expected therefore that a more substantial investment in teaching and preparation will be required to show students how to apply their mathematics to such contexts. Although the mathematics that is needed to be successful in the test of functionality will probably already be known by most students entered for the examination, for many of them it will be necessary to use the material in this resource over a period of time to develop their application skills.

4 Teaching Strategies

4.1 Teaching Functional Mathematics

The method that is proposed for the development of functionality uses similar material to that which is used in the examination: data sheets describing contexts, and mathematically based questions about them. The 'teaching' that has to be done with the material is not teaching mathematics, it is teaching students how to engage in appropriate ways with the information given. The aim of the approach is for students to be able to 'read' context descriptions mathematically, so that they can be functional within that context. In a sense it is teaching them how to read with mathematics in mind, so that they know how to develop an understanding of the context that is mathematical. Of course it is necessary to develop a non-mathematical understanding of the context as well, but this is not sufficient to be mathematically functional.

Part of this is anticipating what types of mathematical questions might be asked. However, this is not the same as trying to guess what exactly the questions might be, and memorise the answers. Such an approach is very unlikely to be helpful, since few if any of the guessed questions will actually be asked. It would also be counter productive, as this memory-based activity will prevent an appropriate engagement with the context.

A more helpful approach involves getting the student to read the data sheet in order to identify the features of the context that could be relevant to the mathematical questions that might be asked. Functionality arises from establishing an effective relationship between the information in the described situation, the mathematics that the person knows that is related to it, and the questions that are asked that require the use of that mathematics on that information.

Over time, as this process is repeated, students should grow in their ability to 'read' contexts so that their mathematical knowledge can be applied appropriately.

This development can be assisted by a gradual change in the teaching.

4.2 Progression in the teaching

1. In the *first stage* as a data sheet is read through by the students, the teacher should support their engagement with it, by suggesting what to notice, and how the information on the sheet could be worked with, discussing with the students the key mathematical elements and possible misunderstandings. Further detail is given below.
2. In the lessons of the *second stage*, the students should be given more opportunity to locate the key elements for themselves before discussion. At this stage, the teacher might suggest general questions to ask
    - What is this telling you?
    - What seems to be important and what may be unimportant?
    - What are the values and quantities and what might be done with them?

   The teacher could also support students as they work with the information, by seeding ideas across groups for example. Then it would be appropriate to discuss student ideas and give additional prompts before the questions are attempted.
3. Finally, in the lessons of the *third stage*, the students should be expected to do all the preparation for themselves, to ask their own questions and work with the information without prompting – and only see with the subsequent questions about the situation whether this was done appropriately, or not. This is good preparation for dealing with information in examinations, but is also the kind of independence in dealing with information that is required for functionality.

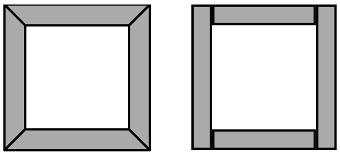
4.3 A Functional Mathematics lesson (first stage)

The specific purposes of each lesson in the first stage of this progression will be two fold: first to enable the students to be able to read and understand the data sheet; and second to enable them to use their mathematics to answer questions about the information on the data sheet.

Before reading the data sheet

For many of the contexts described in the data sheets it may be helpful to talk about the context in general terms before the sheet is actually read. For example, for the data sheet *Radiators* there could be a short discussion about the 'real-life' activity of planning where radiators for a central heating system might be sited, and what sizes they should be, and who would do that planning and how the students think it might be done.

Sometimes it will be helpful to ensure that a discussion covers particular points. For example, on *Frames*, it would be helpful to touch on the shapes of the pieces of wood that make up a picture frame, and to compare a picture frame with a window frame.



picture frame window frame

Helpful preparation for other data sheets may take other forms – for example on *Game of '31'* to actually play the game; for others Internet research may be appropriate eg, *The Solar System*.

Just before moving to read the data sheet, it may be important to remind some students that the focus of attention when reading the data sheet should not only be on the context itself, but also on the mathematics in the context – because the point of the exercise is not to learn how to make picture frames, or design a kitchen (or whatever) but to learn how to read descriptions of contexts with a mathematical focus, since that is what is involved in being mathematically functional. Doing this despite the distractions of engaging information is part of the demand.

Reading the data sheet

The data sheet should then be given out, and time allowed for the students to read the sheet for themselves, with the teacher giving help to individuals as necessary, and also possibly mentioning some aspects of the sheet to pay particular attention to.

Discussion

After reading the data sheet, the students can be asked what they thought, and what they did and did not understand, getting them to ask questions to resolve any difficulties, but also asking questions of them, to make sure that the key mathematical elements of that particular data sheet have been paid attention to. For example, on the data sheet *Kitchen design* attention needs to be paid to the following.

- The dimension on the plan and what this says about the dimensions of each grid square.
- Understanding the real dimensions of the kitchen and how long the dimensions on the plan are.
- What is meant by the width and the depth of the kitchen items and the fact that these are not interchangeable, since if a kitchen unit is turned through 90° just to make it fit the drawers or cupboard doors may not open.
- Practical issues such as leaving enough room for the door to open and not obscuring the window.

If there is an example on the data sheet, this might be gone over carefully, to make sure it is understood, and possibly working through another example (eg, by changing the measurements on the *Frames* example).

On some data sheets it would be helpful for the students to do some calculations based directly upon the data given. For example in *Radiators* to calculate the volumes of rooms, and the number of kilowatts required to heat each room.

It will also be helpful for the discussion to consider what mathematical questions might be asked about the described context. This should be approached as anticipating kinds of questions but not trying to guess exactly what will be asked.

- If questions are anticipated, the data sheet is read appropriately, and unsettling surprises are avoided.
- If questions are guessed, there is an unhelpful temptation to try to remember the answers.

Nevertheless, the anticipation of kinds of questions should include specific examples that the students can actually try, so as to be aware of the mathematics that is likely to be involved. The teacher might have a few of these 'in reserve' in case students find it difficult to formulate the specifics. Also, it will sometimes be necessary for the teacher to direct class attention to aspects of the data sheet as potential sources of questions that the students have not noticed, and to formulate example questions about them.

Some students may remark on the difference between likely questions in a test, and likely questions in the 'real world'. Examples of each could then be collected, to find out if the actual questions, which reflect the kinds of questions that will be asked in the examination, correspond more closely to their ideas about one or the other.

Answering the questions

Depending on timings, and other considerations, answering the questions of the item may be done during the same lesson, or in a subsequent lesson. This really does not matter, since in the examination some data sheets will be given as pre-release material, but others will be available only during the examination itself. There is a benefit to sometimes leaving time between the reading of the data sheet and answering the questions on it, yet at other times answering them straight away.

The questions should probably be attempted by the students without discussion with the teacher, even though sometimes there is additional data in the questions that will need to be incorporated into their understanding of the context. However, individual queries should be clarified, and possibly with the whole group. For example, Question 2 in *Radiators* asks for "the most sensible choice", which may need to be explained to some students. Some students may also need to be reminded that the questions draw on information in the data sheet, and that a quick answer that does not refer back to the sheet, but is based on their own understanding, will not be enough.

After answering the questions

As well as seeing whether their answers are correct, and discussing successes and errors, it is also valuable to discuss with the students what the challenges were in knowing what to do, in order to bring out the relationship between engagement with the described situation and the mathematical action. In other words, there could be a reflection on what aspects of their initial understanding – through their reading and thinking about the context – proved to be helpful. There might also be a discussion about the nature of mathematics in the real world, and how such considerations should affect the use of the data and the reading of the data sheet.

4.4 Functional Mathematics lessons (Second and third stages)

Examples of the kind of lesson described above should be followed by other lessons that gradually cede responsibility for making mathematical sense of situations to the students themselves, as described in the section 'Progression in the teaching'. At the end of that sequence, students should be able to operate independently with mathematically related information, without the teacher's support.

A further possibility during these stages is to consider other contexts (found for example by looking on the Internet for suitable sources, or working from suggestions made by the students themselves) and explore with the students what possible mathematically related questions could be asked about them – in the spirit of 'reading' contexts with mathematics in mind, rather than trying to anticipate possible examination questions.

4.5 Preparing for the examination

When the school receives the pre-release material for the actual examination, in the form of two 'data-sheets', it will be helpful for the teacher to support students in their preparation. The best form for this involvement is probably not at the level described in the 'first stage' lesson, but closer to that in the 'second stage' – having students take the initiative with the first readings of the pre-release material, but then engaging them in discussion, and ensuring that the key points have in fact been noticed, and rehearsing some of the likely mathematics – but not trying to guess the questions!

5

The CD-Rom of Tasks

5.1 Introduction

The enclosed CD-Rom contains 30 functionality tasks consisting of a data sheet and a number of questions. These are presented alphabetically in PDF format. The CD also contains a PDF copy of this document.

All files on the CD may be copied for use in centres for the intended purpose only and must not be reproduced for any other reason, including commercially.

6

Commentaries

6.1 Introduction

For ten of the tasks in the resource, a detailed commentary is provided on the following pages. They describe features of the data sheets and comment on the questions which follow.

6.2 Bricklaying

There is an apparent simplicity in the way bricks can be laid on top of each other and one of the difficulties with the topic may be that many students are surprised by how much mathematics there is in the laying of bricks. There are many patterns that can be followed with implications for how many bricks will be needed, the costs and how they look.

Initial data

Bricklaying is a technical subject and there is first a need to come to terms with the language of 'stretcher' and 'header' as well as the different types of 'bond'. Demonstration with actual bricks or brick substitutes such as blocks might help in this. Another feature that students may not be familiar with is the way bricks are bought: the price is quoted in so much per thousand bricks, though they are bought in units of 500. Some practice in this along the lines of, 'How many would you have to buy if you wanted *x* bricks and how much would they cost?' would be useful.

Commentary on questions

1. The three parts to Question 1 are at a simple level of demand to check that the way bricks are priced and sold has been understood.

2. The two parts of Question 2 are assessing whether the stretcher bond style of laying bricks is understood. This should be straightforward using the data sheet.

3. More demanding than the previous two questions, this requires an understanding of a particular way of laying bricks – English bond. Earlier experience with bricks or blocks would be useful here.

4. This question introduces a new aspect of English bond, namely that the structure does not allow all bricks to be seen. Students need to recognise which parts of the given diagram are 'hiding' other bricks. This can be deduced from the data sheet. The wall is set between two posts to imply that the sides cannot be seen.

5. This extends the idea of unseen bricks by pointing out that cheaper bricks can be used for those that are not seen. The problem set demands a more sustained engagement with the practicalities of buying bricks.

6.3 Codes

The use of an encryption method based on square numbers to create or resolve a code presents issues of reasoning and data handling. Some students may have experience with codes, but it is unlikely that it will have included a code of this kind. Nevertheless, those students may be both the most motivated and the most inclined to jump to conclusions about how the code will work.

In the questions there is a further demand of explanation, not only to understand what is going on but also to make clear statements about it. Some students may find this comparatively challenging.

Initial data

The data sheet starts with a coded message and it may not be clear, when following the sequence to decode the message, where the key (32 squared) came from. It may be helpful therefore to get students to do some encryption of this kind – starting with a square number and a message in columns. It may be useful to consider (here or later) why smaller two-digit square numbers are not suitable for this code.

Commentary on questions

1. The first question should be relatively straightforward, especially if the study of the data sheet involved some code making and breaking.

2. This is another straightforward question which requires just a little more understanding of the encryption process by showing what a message might have looked like before encryption.

3. Some students may try to answer this by considering the features of 38, without squaring it first, but once they have found that 38 squared is 1444 most will recognise the difficulty – although they may find it difficult to express clearly in words.

4. This question requires an explanation, and the concept to be explained is quite a subtle one. It is unlikely that many will go so far as to work out how many different possible arrangements of columns there are (720 and 24 respectively) but some might.

6.4 Digital prints

Digital photography is increasingly common and this item uses it as a context for reading tables of prices and integrating information to find total costs. The first two questions are straightforward. Questions 3 and 4 are more challenging with the introduction of additional data for Question 4.

Initial data

In relation to the context it would be useful to talk about getting photographs printed and let students, who have done this either by e-mailing photos to companies or taking a memory stick to a supermarket or shop, talk about how the costs were structured.

More specifically the whole data sheet should be read through with the students including the table and table headings and potential sources of misunderstanding should be discussed. Some students may fail to notice that the costs are given per print and not for the full set of prints. Prices are related to ranges of quantity. It is important to ask students specific questions about the print costs and postage and handling charges for different numbers of prints to check their understanding of what is presented in the table. The last row of the postage and handling charges table may require some explanation for some students since it gives the postage cost for each additional 50 prints or part of 50.

Commentary on questions

1. This is straightforward, requiring the students to find the cost per print when 75 prints are wanted; this will be 12p per print × 75 prints.

2. This question requires a little more in that the price for 101 prints and then the price for 100 prints are needed which then have to be compared.

3. The first part of this question introduces the use of the postage and handling charges. The costs of the 75 prints alone should already have been calculated for Question 1 then the price for the postage and handling is required. Part (b) is more difficult involving more steps. Students have to start from the unit price per print for 800 prints, add in the postage to get the total cost before converting back to unit cost per print.

Additional data for Question 4

Another table is now introduced for another print company that includes the cost per print and postage and packing. Some students may need help to understand the information in the table.

4. This question requires the student to compare the price of 80 prints for each company. This requires the use of information in the new table as well as the previous two tables. Although the calculations themselves are not difficult, the use of information from all three tables might be a challenge for some students.

6.5 Game of '31'

Games (often of a mathematical nature) have existed throughout history in probably all cultures. The issue of whether a game is a matter of skill or 'just luck' or some combination of the two is a tantalising one. The game of '31' is an easily accessible activity based on simple arithmetic. The arithmetic lends itself to analysis so that strategies can be formulated to increase the likelihood of winning or even guarantee a win, if carefully followed. Forming a strategy to win requires some analysis of the arithmetic, which is more demanding than just playing the game, but it remains well within the capability of most people. Variations on the game can make it harder or easier.

Initial data

By far the best way to engage students with this particular context is for them to play the game of '31' for themselves. This enables them to understand the procedures for playing and offers a base from which to discuss possible strategies for winning. Initially, this could be a few games between the teacher and the class as a whole, where the teacher (knowing a winning strategy) always wins the game. This can be a good stimulus to students since it will convince them that there is a strategy for winning and motivate them to find it; they can then set about playing a few games between themselves in pairs and trying out the possibilities suggested on the data sheet.

Variations on the game are suggested. Strategies for winning will change and may even disappear entirely as the constraints on play are altered. Inventing and playing variations on the game will help develop a more generalised understanding of the arithmetic structure.

Commentary on questions

1. An elementary question to see if the rules of play are understood. It should not cause any problems.
2. A development of Question 1, inviting a strategy that recognises that a gap of at least 7 always needs two turns of play to bridge it. This is still an easy question that can be thought through by envisaging playing the game.
3. This is harder than Question 2 in that it requires a more formal articulation of the thinking done there. Not only is mathematics required, the ability to put it into words is also necessary.
4. This builds further on the ideas from Questions 2 and 3. It requires students to construct a logical development from the recognition that 24 is the significant number that precedes 31, so what is the significant number that precedes 24?
5. To solve this question, students need to carry the thinking built up over the previous questions and adapt it to a variant of the game of '31', where the target is now 42 and a different set of numbers can be used. This is likely only to be accessible to students expecting a D grade or higher.
6. Finally the game is changed completely and a new strategy has to be developed, though the question indicates an approach.

6.6 Hair Salon

This item uses the context of hair colouring in a hair salon and requires students to extract the required information from the tables. Students need to be able to work confidently with simple fractions such as $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$, calculate fractions of numbers, such as $\frac{1}{4}$ of 20 and simplify fractions.

Some everyday contexts may have relevance more to one gender than another and this is possibly one that appeals to girls more than boys. There will be others with the opposite appeal.

Initial data

It would be useful to ask students' to discuss their experience of visiting a hair salon and if they know anything of the process of hair colouring. The mixing process is often not seen first hand, but students may have experience of the hairdresser going away to mix colours. Some students will also have experience of colouring their own hair using a ready mixed colour from a chemists or a supermarket.

The data sheet shows two tables. The first table is simply a list of the required amount of colour for differing lengths of hair and as expected the longer the hair the more colour is required to cover it. The other table shows how the main colours of black, brown, red and blonde can be mixed to create other hair rinse colours such as dark brown and auburn. Ask students specific questions about how much colour mixture is required for different lengths of hair and for different hair colour to check that they have understood the information in the tables and can work with it.

Commentary on questions

1. This simply requires the student to read from a table.
2. The second question again only uses the information presented in the first table, although this time there is a further step.
3. Question 3 requires the use of both tables, so is a little more challenging.
4. This question introduces a very small amount of new data, which is that each main colour comes in a 100 ml tube.
5. In the final question information from the two tables is used, and the outcome of calculations has to be converted into fractions of a tube. This final step may be best approached by way of an interim answer of the amount in millilitres of each colour needed.

6.7 Heart rate

This activity involves the use of formulae, graphs and percentages in the context of heart rates and the appropriate exercise levels for individuals. Students are likely to have some understanding of the context of the questions – for example, that heart rate increases in the short term with exercise, and that a structured programme of exercise can increase a person's overall fitness level.

The combination of the context and the nature of the mathematical skills required makes this item demanding, especially towards the end of the activity.

Initial data

The two formulae presented in the data sheet might require some careful calculator work with students (particularly the second formula), in order for them to be able to use them reliably. The precise meaning of *maximum heart rate* compared to just *heart rate* might need some careful teasing out. A graph showing MHR versus age for both of the original formulae is also introduced. Some support in interpreting this graph may be necessary.

To deal with the training programme aspect, students will also need to be comfortable with the meaning of percentages and be able to do simple calculations with percentages of the form x% of y, and to work out x/y as a percentage.

Commentary on questions

1. This is straightforward, requiring only the correct use of the level table.

2. This question requires the application of two formulae. The first formula is simple and this part should be accessible to most students. The second formula is more complex, however, and problems might arise with the sequence of operations.

3. In part (a) students need to understand that the two methods are giving the same value where the lines cross. The answer is not exact and so sensible estimates are acceptable. The level of demand increases in part (b) since a written explanation, comparing the two methods for the relevant age group, is required. This will be demanding for many students.

4. Both parts of this question are two-step in that students have first to use a formula to calculate the MHR and then apply to their answer the percentage for the appropriate level. Some may find this difficult to sustain.

5. This requires students to calculate the MHR using both methods and then use the simple method as a base for working out the percentage difference in the two values they have found. There may be confusion over identifying the base number from which to calculate the percentage.

6.8 Radiators

This item, based on actual information used by plumbers, requires an engagement with diagrammatically presented information about room dimensions, and further information presented as a formula, in a table, and in text form. Questions are asked that involve extraction of data from these sources, at first separately and later in combination. The other mathematics called on is simple calculations and substituting values into a formula, but finally there is a multi-step question involving inverse operations.

Initial data

There are three information sources in the initial data, and each of them needs to be considered. The plan should be straightforward, but there may be uncertainty about what is being described in the second source. Some background on what a kilowatt is may be needed, and the question of how to work out the volume of a room may need some attention. The significance of the 'north facing outer wall' should also be raised, and there the meaning of 'back porch' may have to be considered. Another issue to look at is the watts and kilowatts relationship, since the formula of the second information source gives an answer in kilowatts, but the chart of radiators gives the heat output in watts.

There might also be a discussion about the size of radiator to put in a room. What would be the effect of a radiator that is too big or too small? What is acceptable?

Commentary on questions

1. Part (a) is straightforward – on a calculator - requiring the calculation of 3.5 × 3.6 × 2.4 × 0.04. However, some may forget to factor in the 0.04 and just give the volume. In part (b) the 'north facing outer wall' aspect has to be factored in and the students who do not notice this will be a little confused, since the room dimensions are the same.

2. This question is of medium demand because there is a calculation (24.192 × 0.04), a conversion (kilowatts to watts) and a requirement for interpretation – do you go for the one above or the one below? Not being able to engage with 'most sensible choice' in this context may be a barrier for some, and it is hard to argue that it is wrong to say that a 065d should be used 'to save money'.

3. Question 3 introduces the idea of radiators being 'made-to-measure' in different sizes. This is a more demanding question, and different students could approach the question in different ways. One way is to work out the heat output of each 100 mm of 600 mm high double radiator, a 'direct proportion' method. Different radiators give slightly different answers, but the variation is small. Dividing any of these into 3200 gives an answer around 1.86 metres, so the 1900 mm radiator is needed.

An alternative approach is to use proportionality on one example, saying for example that 3200 watts is about 1.55 times more than the 1200 mm radiator gives, so a radiator that is at least 1200 × 1.55 (= 1865 mm) would be needed ie, a 1900 mm one.

A third approach is to see that 3200 watts is just a little bit more than the combined heat output of the 700 and 1200 double radiators, and thus reason that a 1900 mm radiator would do the job.

6.9 Rainforest facts

The item focuses upon the rainforests of the world, their destruction for a variety of purposes and the effects of this destruction upon certain species of plants and animals. The item could be used in tandem with other resources that derive from issues such as conservation and climate change. The mathematics is initially concerned with basic arithmetic based upon data extracted from the data sheet and the reading and understanding of some large numbers. However an understanding of percentages is also required – to be applied to additional data provided in one of the later questions.

Initial data

Students could be encouraged to engage with the data by looking at maps or a globe to see where the rainforests are, in particular the location of Sumatra and Borneo. The questions are initially very straightforward, extracting data from the text and processing it. The later questions require some understanding of ratio and proportion.

Commentary on questions

1. Part (a) requires no more than selecting the correct piece of data from the text and multiplying by 7
2. For part (b) the value extracted needs to be multiplied by 24 and by 7 to get the weekly figure.
3. Part (a) requires the identification of two percentages and the subtraction of the smaller from the larger. Part (b) is more subtle: 15% has reduced to 7% so the percentage of the 1950 rainforest that has been lost is 8/15 × 100%.
4. Here the locating of the pieces of data may prove more difficult than the calculation.
5. This question extrapolates some of the given data. Compound percentages can be tricky, though the table here supports the necessary calculations. The greater challenge may be for students to understand how the table is doing this and to make appropriate roundings.
6. There are a number of steps to the solution to this question, and mistakes may arise from the challenge of sustaining engagement across them all.

6.10 The Solar System

The solar system is a familiar context for students, and many of them will have some knowledge about it already, although probably not in as much detail as the data sheet.

Initial data

The data given on the solar system has some challenges for understanding, both in terms of the language used, and in terms of the mathematical relationships employed.

The notions of orbit, ratio of orbit and period of orbit need to be well understood, and the idea of using the Earth as the reference point for measurements is also likely to cause issues for some students.

If students raise the 'Pluto question' (as the former 9th planet) it might be discussed (What is a planet? See perhaps http://en.wikipedia.org/wiki/Definition_of_planet)
- or dismissed, by saying that one of the questions to be answered is about Pluto.

Commentary on questions

1. This is a straightforward question that involves a small amount of interpretation of the given data.

2. This may be a little more demanding, as it involves the addition of decimals, and coping with the notion of 'Earth masses'. The fact that the masses of Mercury, Venus and Mars add up to more or less the same as the Earth is intriguing, and may lead to some students questioning it.

3. A simple multiplication (0.241 × 365.3) is required, using information readily taken from the data sheet, but interpreting the sentence about Earth days and Mercury's orbit is quite tricky. It may be helpful to ask the intermediate question: 'How many Earth *years* does it take for Mercury to orbit the sun?'

4. This is structurally the same as Question 3, but here the challenge is in translating back – from distance from the sun to radius of orbit – in order to know what information in the table is required. One of the issues for some students may be that many calculators will render the answer in standard form.

5. The challenge in part (a) is in interpreting the graph correctly, so that the appropriate information is found on the data sheet. The actual plotting of the point is straightforward – so long as students are comfortable about estimating.

Part (b) requires students to be confident in making a statement about an approximate relationship. Some students may not want to say anything about the relationship because it is not clear cut. Others may know what they see in the relationship but struggle to find words that they are comfortable with.

Additional data for Question 6

The introduction of Kepler's third law could be challenging for many students, at least initially, but the mathematics of it is not complex and should be within the range of most. The meaning of the equation is the heart of it and the example is given to help to show what it means.

6. All that is required for this question is that 39.5 is cubed, and then the square root of that is found, but the challenge of course is recognising that. If students need support, then a stepped approach might be suggested.

What is R? What is R cubed? What therefore is P squared? What is P?

6.11 Transport Issues

This context is likely to be one that students will have considerable familiarity with, both from their day-to-day experiences and from the regular coverage in the media.

Initial data

Information is presented through the results of a survey of people's opinions on the relevant issues. Students will need to be comfortable with the reasoning behind the use of such surveys, how they are carried out, how the results of them should be interpreted, and how the findings might be used to inform policy. The mathematical activities that arise from the survey responses include calculations of actual numbers of respondents in certain categories, representations of the data using simple graphs to indicate changes over time, and written interpretations of the data.

There are possibilities for class discussion of the findings of the survey as well as consideration of the specific characteristics of it, such as:

- Why does the sample only include the over 15s?
- Why do the percentages in A and C add to more than 100 in each year, but those in B do add (approximately) to 100?
- Were the same respondents used in 2003 and 2004?

Commentary on questions

1., 2. These both ask for readings from the tables and should be straightforward.

3. This is more difficult and requires students to take carry out the intermediate step of working out the combined percentage in the two categories and to then go on to convert that into an actual number of respondents.

4. The mathematics (0.5% of 2016) is relatively straightforward but the embedded context may mean that arriving at this calculation is a difficult set of steps for many students.

Additional data for Question 5

New information relating to the cost and times of the London congestion charge is presented.

5. Confident students should be able to make sense of this problem, but it is likely that a number of unsupported students will be unsure of what exactly they are being asked to do.

6. Many students find questions like this difficult because they first have to be clear in their minds what they are being asked, and then distil quite a lot of information into a short written explanation. If work with the original data on combining related survey categories has taken place, then combining the two 'support' and the two 'oppose' categories for this question should not be such a difficult step.

7

Answers

Question	Number	Answer
Bacterial growth	1(a)	Any value over 1300 but less than 1500
	1(b)	Any value over 2750 but less than 3000
	1(c)	One hour approximately
	2	At 6 hours the population is (approximately) 750, and at 7 hours the population is at (approximately)1500 **or** At 7 hours the population is (approximately) 1500, and at 8 hours the population is at (approximately) 3000
	3	2560 (5×2^9)
	4	32 times larger (X has population 2^{14}, and Y has population 2^9)
Bookclub	1(a)	£14.99
	1(b)	£22.49
	2	Bookish
	3	Blockwells is cheaper by 10p
	4	Yes, it looks like it would be worth joining a book club – both ABC and PAGES work out at £20 per month for two books (using the costs of the two books given). All of the book shops cost well over twenty pounds for the pair.
	5	£4

Question	Number	Answer
Bricklaying	1(a)	£247.50
	1(b)	9 pallets
	1(c)	£2227.50
	2(a)	360
	2(b)	18
	3	31
	4	12
	5	£9175
Calendar	1	1968 and 1972
	2	2nd of July
	3	Only the century years that divide by 400 are leap years so 1600 is a leap year but 1700 is not.
	4(a)	2002 Wednesday, 2003 Thursday, 2005 Sunday, 2006 Monday
	4(b)	2010
	5	April, May, June **or** September, October, November. Also allow December, January, February **or** January, February, March for February in a leap year.

<table>
<tr><th>Question</th><th>Number</th><th>Answer</th></tr>
<tr><td>Codes</td><td>1</td><td>Do not swallow pips</td></tr>
<tr><td></td><td>2</td><td>
<table>
<tr><th>2</th><th>4</th><th>0</th><th>1</th></tr>
<tr><td>d</td><td>e</td><td>i</td><td>r</td></tr>
<tr><td>i</td><td>k</td><td>b</td><td>a</td></tr>
<tr><td>o</td><td>w</td><td>t</td><td>e</td></tr>
<tr><td>k</td><td></td><td>r</td><td>o</td></tr>
</table>
</td></tr>
<tr><td></td><td>3</td><td>38^2 is 1444. The repeated '4's mean that when coding or decoding a message, you would not know in which order to place the three columns that each have '4' at the top.</td></tr>
<tr><td></td><td>4</td><td>B There are fewer columns, so the number of possible arrangements will be fewer.</td></tr>
<tr><td>Coffee</td><td>1</td><td>$\frac{20}{100} = \frac{1}{5}$</td></tr>
<tr><td></td><td>2</td><td>24%</td></tr>
<tr><td></td><td>3</td><td>13 million</td></tr>
<tr><td></td><td>4</td><td>1% (1.14…%)</td></tr>
<tr><td></td><td>5(a)</td><td>500</td></tr>
<tr><td></td><td>5(b)</td><td>There are people who do not drink any coffee. When working out the mean for coffee drinkers alone, the number of people is actually less than the 60 million used in part (a). Hence, this second mean will be more than 500 cups in a year.</td></tr>
<tr><td></td><td>6</td><td>The proportion has decreased over the period since the amount received by the farmers has stayed approximately the same (at 20 p approximately), whereas the price in the shops has increased (from approximately 125 p to 180 p).</td></tr>
<tr><td>Digital Prints</td><td>1</td><td>£ 9</td></tr>
<tr><td></td><td>2</td><td>£ 2.91</td></tr>
<tr><td></td><td>3(a)</td><td>£ 10.99</td></tr>
<tr><td></td><td>3(b)</td><td>6.69 p (rounded 7 p)</td></tr>
<tr><td></td><td>4</td><td>Pixyprints company is cheaper by £ 1.19</td></tr>
</table>

Question	**Number**	**Answer**
Energy labels	1(a)	LEX BR20
	1(b)	325
	1(c)	£9.50
	2(a)	Coolmaster C, Bravo B, Vortex A
	2(b)	Zen
	2(c)	4 years
Farm animals	1	1 700 000
	2	9 139 000
	3	1999
	4	995 000
	5	170 million Compared to the year before (2003) the number of table fowl has increased by 3000 or so (ie, 3% or so), whilst the number of laying fowl has also increased, by 1% or so. It seems reasonable to assume that the total number of fowl will also increase by a small percentage, say 2%, which gives 3000 more compared to 2003. Hence the nearest of the available answers is 170 million.
Frames	1(a)	576 cm^2
	1(b)	144 cm
	1(c)	6 cm
	2	24 cm
	3	140 cm
	4	289 cm

Question	Number	Answer
Game of '31'	1	5
	2	Megan can choose any of 1, 2 or 3
	3	Explanation that recognises that 24 to 31 needs two goes since 6 is the largest available number to add on and the gap is 7.
	4	5 (to get to 17)
	5	Emma. 33 is 9 less than 42 but the largest number available is 8, so it will need two goes. 34 is 8 less than 42 and so 42 can be reached in one go from 34 by choosing 8.
	6	Whatever Aaron chooses from the list, Zoe can choose a number that makes the two numbers chosen add up to 10. For example, if Aaron chooses 9, Zoe can choose 1. In this way Zoe can make the total reduce from 50 to 40 to 30 to 20 to 10 and then, finally, she can make it zero and win the game.
Gold	1(a)	$\frac{20}{24} = \frac{5}{12}$
	1(b)	18 karat
	2	154.4 g
	3	0.125 g
	4	2000 000 kg
	5	£ 11.44
	6	7
Hair Salon	1	50 ml
	2	10 ml
	3(a)	Brown, red, blonde
	3(b)	Brown 10 ml, red 20 ml, blonde 10 ml
	4	2/5
	5	Black 3/20, brown 1/20

Question	Number	Answer
Heart rate	1	Fitness level (Fat burning)
	2	195, 189 (188.675)
	3(a)	40 to 44
	3(b)	For younger people (anyone under 40 or so), the simple method gives a higher MHR estimate than the more accurate method. So Eshan's calculation overestimates the MHR for his group.
	4(a)	130 (70% of 185.25 = 129.675)
	4(b)	148 (80% of 185.25 = 148.2)
	5	1% increase (1.55/170 × 100% = 0.91%)
Internet advertising	1	£100
	2	£625
	3(a)	7p per click
	3(b)	Banner for a full year (5p per click)
	4	20%
	5	1 large tower advert for a month, 1 banner advert for a month and 1 banner advert for 3 months.
Kitchen design	1	Washing machine
	2	90cm
	3	3
	4	7
	5	A = double cupboard unit, B = small 3-drawer unit
	6	C – oven, D – washing machine, E – fridge freezer

Question	Number	Answer
Nines	1(a)	0.9999
	1(b)	99.99%
	2	2N5
	3(a)	95%
	3(b)	72 minutes
	4	5 g
Plastic codes	1	May 2006, heat resistant, unsuitable for food use, recyclable type 6
	2	The month is wrong – it should be between 1 and 12 but is 14 The recyclable code is wrong – it should be between 5 and 9 but is 3
	3	The first and last do not match their checking digits
	4	0609135 X
	5	No For example, both 127 and 172 divide by 9 with remainder 1. (The rule for divisibility by 9 is that the digit sum is 9 and this sum does not change if the digits are swapped.)
Premium Bonds	1(a)	1 456 387
	1(b)	8 488
	2	£ 1.75 million
	3	20%
	4	£ 1300
	5	3.5%
	6	80%

<table>
<tr><th>Question</th><th>Number</th><th>Answer</th></tr>
<tr><td>Radiators</td><td>1(a)</td><td>1.2096 (or 1.2)</td></tr>
<tr><td></td><td>1(b)</td><td>0.3024 (or 0.3) [or 25% of answer to part (a)]</td></tr>
<tr><td></td><td>2</td><td>310 d</td></tr>
<tr><td></td><td>3</td><td>1900</td></tr>
<tr><td>Rainforest facts</td><td>1(a)</td><td>959</td></tr>
<tr><td></td><td>1(b)</td><td>3 696 km^2 per week</td></tr>
<tr><td></td><td>2(a)</td><td>8%</td></tr>
<tr><td></td><td>2(b)</td><td>53.3% (rounded 53%)</td></tr>
<tr><td></td><td>3</td><td>195 000 km^2</td></tr>
<tr><td></td><td>4</td><td><table>
<tr><th>Year</th><th>Projected Population</th><th>Estimated Decline</th></tr>
<tr><td>2007</td><td>60 000</td><td>4800</td></tr>
<tr><td>2008</td><td>55 200</td><td>4400</td></tr>
<tr><td>2009</td><td>50 800</td><td>4100</td></tr>
<tr><td>2010</td><td>46 700</td><td>3700</td></tr>
</table></td></tr>
<tr><td></td><td>5</td><td>37.9 days (rounded 38 days)</td></tr>
<tr><td>Shoe sizes</td><td>1</td><td>$\frac{2}{3}$ inches</td></tr>
<tr><td></td><td>2</td><td>36</td></tr>
<tr><td></td><td>3</td><td>24 cm</td></tr>
<tr><td></td><td>4(a)</td><td>42 and 43</td></tr>
<tr><td></td><td>4(b)</td><td>$8\frac{1}{2}$ and 9</td></tr>
</table>

Question	**Number**	**Answer**
Speed check	1	Vehicle types B and E
	2	Open areas (both single and dual carriageways)
	3	79 mph
	4	45 feet in a half-second = 7200 × 45 feet in an hour = 324 000 Then this in mph is 324 000 ÷ 5280 = 61.4 mph ie, over 60 miles per hour
	5	Yes 1.5 miles in 1.5 minutes is the same speed as a mile a minute ie, 60 mph. So the driver has broken the 50 mph speed limit.
Steep hill gradients	1(a)	Uphill
	1(b)	40 meters
	2	1:6
	3	4%
	4(a)	The steepest is 18%
	4(b)	The least steep is 1:12
	5	25%, 1:4

<table>
<tr><th>Question</th><th>Number</th><th>Answer</th></tr>
<tr><td>Sunshine Hotel</td><td>1</td><td>3</td></tr>
<tr><td></td><td>2</td><td><table>
<tr><th></th><th>Sunday</th></tr>
<tr><td>Mike Harvey</td><td>7 am - 3 pm</td></tr>
<tr><td>Shirley Jones</td><td>11 am - 6 pm</td></tr>
<tr><td>Alan Marks</td><td>3 am - 11 am</td></tr>
<tr><td>Jennifer Bartlett</td><td>12 midnight - 3 am</td></tr>
<tr><td>Sarah Parker</td><td>6 pm - 12 midnight</td></tr>
<tr><td>David Williams</td><td>3 pm - 11 pm</td></tr>
</table></td></tr>
<tr><td></td><td>3</td><td>18 hours</td></tr>
<tr><td></td><td>4</td><td>£115.92</td></tr>
<tr><td></td><td>5</td><td>Sarah Parker or Jennifer Bartlett</td></tr>
<tr><td></td><td>6</td><td>Because if he did start at 12 noon he would not have had the required 12 hours off between the two shifts (both of which are 5 hours or more long).</td></tr>
<tr><td>The Solar System</td><td>1</td><td>Jupiter (whether mass or volume is considered)</td></tr>
<tr><td></td><td>2</td><td>2</td></tr>
<tr><td></td><td>3</td><td>88 (or 88.0373)</td></tr>
<tr><td></td><td>4</td><td>4 497 million kilometres (or equivalent, and allowing for different rounding)</td></tr>
<tr><td></td><td>5(a)</td><td>A point on the graph roughly corresponding to the coordinates (9.41, 56)</td></tr>
<tr><td></td><td>5(b)</td><td>Something along the lines of 'the bigger the diameter, the more moons there are'</td></tr>
<tr><td></td><td>6</td><td>248 (or 248.25365 etc,)</td></tr>
</table>

Question	**Number**	**Answer**
The world's population	1	1920
	2	416 million
	3	20%
	4(a)	15.24 million
	4(b)	15.22 million
	5	Estimate A → The population of the world will continue to increase but not as quickly as it has in the last 50 years. Estimate B → The population of the world will continue to increase more or less as it has in the last 50 years.
Trains	1	11:47
	2	11 minutes
	3	Every half hour
	4	14:17
	5	22 minutes
Transport Issues	1	2016
	2	Bus (down 7 per cent)
	3	1500 (70% of 2102=1471.4, then rounded to nearest 100)
	4	10 (0.5% of 2016=10.08)
	5	73 pence per hour (£8 ÷ 11 hours = 0.7272…)
	6	A statement such as: 'The number who support congestion charging has declined (down 3%) whilst those who oppose it have increased (up 3%).' **or** 'Overall the survey indicates that there has been a drop in support and an increase in opposition to congestion charging in 2004 compared to 2003.'

Question	Number	Answer
Unusual measures	1	218 m or 218.4 m
	2	53 or 53.4
	3	6 times or 6.276 or similar
	4	821 or near – different approaches to approximation could give a number of different answers in the range 800 - 850
	5	Almost 46 years (2500000 ÷ 150 ÷ 365 = 45.66)
	6	553 km
	7	Approximately 28